Gauge Capability Studies for Attribute Data

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A gauge repeatability and reproducibility (R&R) method is developed to assess the capability of a measurement system. Although gauge capability studies have received substantial attention, few studies have investigated attribute data despite their wide application in industry. The primary aim of this research is to develop a procedure, based on the generalized linear model, to evaluate the R&R of a measurement system for attribute data. To calculate repeatability of a system, the procedure integrates the iterative weighted least squares (IWLS) method and deviance analysis. The proposed procedure is applied to an inclusion measurement system to verify its adequacy to model the process capability. Copyright © 2007 John Wiley & Sons, Ltd.

Received 15 November 2005; Accepted 12 July 2006

KEY WORDS: repeatability; reproducibility; generalized linear model; attribute data; gauge study

1. INTRODUCTION

Repeatability and reproducibility (R&R) is adopted for analyzing measurement variation of a gauge (repeatability) and determining the variation of measurements by operators (reproducibility). Gauge capability analysis is a crucial element to improve a measurement system and is an integral part of efficient process control. With numerous industries utilizing automated manufacturing processes and high-speed automatic inspection equipment, gauge capability studies are essential to determining the components of variation in measurement processes, see, e.g., Borror et al.1, Stravinski and Sype2, Kim et al.3 and Burdick et al.4. Many international standards, such as ISO/TS 16949, ASTM F 1469 and ASTM E 691 include R&R analysis in the clauses. In applying the ISO/TS 16949 standard to the measurement system analysis (MSA), Dasgupta and Murthy5 demonstrated methods to improve gauge R&R through a case study.

Since Mandel6 described a method for calculating R&R over three decades ago, there have been plenty of studies about this discipline. Montgomery and Runger7,8 improved conventional gauge capability analysis using experimental design approach. Their studies employed alternative estimation procedures of the variance components that were adopted by ISO/TS 169499 and MSA reference manual10. Although more complicated approaches have been developed by other researchers (such as Burdick et al.11 and Daniels et al.12), practitioners prefer using simple methods to calculate gauge capabilities. The American Society for Testing and Materials (ASTM) issued E 69113 to define standard procedure for performing inter-laboratory studies

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to determine R&R of test methods. ASTM E 691 standard is now widely adopted in industry despite its straightforward calculation procedure.

Although many gauge capability studies exist, few research papers have discussed cases of attribute data, regardless of whether they are common in industry. For example, defects of silicon wafer, number of non-conformities on a printed circuit board (PCB), number of bright spots on an TFT-LCD panel, and the amount of inclusion content of steel are all cases to which conventional gauge capability analysis does not apply. Boyles\textsuperscript{14}, Ranjan \textit{et al.}\textsuperscript{15} and Roy \textit{et al.}\textsuperscript{16} considered misclassification rate—acceptance of a bad unit or rejection of a good unit—and then modified the gauge capability model and process control limits. Hirschler\textsuperscript{17} modified ASTM E 691 standard in revising the R&R calculation method for binary experimental data. Van Wieringen and van den Heuvel\textsuperscript{18} modeled the outcome of an MSA experiment of binary measurement by using a latent class model and EM algorithm to measure between-laboratory consistency. For bounded ordinal data, De Mast and van Wieringen\textsuperscript{19} developed an approach for the measure of agreement between two appraisers, known as Cohen’s kappa statistic in ISO/TS 16949. Mandel\textsuperscript{20} proposed a method suited to pass/fail and continuous data for evaluating R&R in inter-laboratory studies.

Generalized linear models (GLMs) are effective in analyzing attribute data and have been applied to monitor processes, e.g., Hansen and Thyregod\textsuperscript{21}, Jearkpaporn \textit{et al.}\textsuperscript{22}, Tu and Piegorsch\textsuperscript{23} and Skinner \textit{et al.}\textsuperscript{24,25}. Such a wide application indicates that GLM can enhance the effectiveness of gauge capability analysis in environments in which variables are exponential family distributions.

This study presents a novel model for calculating R&R for attribute data, utilizing GLM as the basis of a new algorithm. An alloy-manufacturing case is used to demonstrate the feasibility of the proposed model.

2. MODEL DEVELOPMENT

In this study, $Y_{ij}$ is the measurement result for $j$th specimen in laboratory $i$. $Y_{ij}$ can be posited as a very general model

$$Y_{ij} = \mu_Y + M_i + S_j + E_{ij}, \quad i = 1, \ldots, m, \quad j = 1, \ldots, s$$ (1)

where $\mu_Y$ is the true average value of measurement results in all laboratories, $M_i$ is the fixed effect by which laboratory $i$ deviates from this true value, $S_j$ is the fixed effect by which specimen $j$ deviates from this true value, $m$ is the sample size of laboratory, $s$ is the sample size of specimen, and $E_{ij}$ is the random error for $j$th specimen in laboratory $i$.

All laboratories are assumed to have the same level of variability when following the specified repeatability conditions. This assumption is not always true. When a laboratory tests a particular material for a short time, the likelihood that this assumption is true increases. These measurement results incur repeatability variance and reproducibility variance.

The repeatability variance is a measure of within-laboratory variability of each laboratory. Reproducibility variance among individual test results obtained in different laboratories is the sum of within-laboratory and between-laboratory variances of the laboratory means. The square root of the repeatability variance is repeatability standard deviation, and the reproducibility standard deviation is the square root of reproducibility variance. These definitions can be found in ASTM E 691\textsuperscript{13}.

To obtain within-laboratory variance, a GLM (Section 2.1) was employed to generate estimates ($\hat{Y}_{ij}$) of measurement results. To acquire and test the regression parameters for a laboratory, iterative weighted least square (IWLS) and deviance analysis were integrated to develop the repeatability function (Section 2.2). To generate between-laboratory variance to construct a reproducibility function, reproducibility variance is calculated. This calculation is discussed in Section 2.3. The evaluation of gauge capability was formulated in Section 2.4 and a gauge R&R model was presented in Section 2.5.
2.1. Generalized linear models

Ordinary least squares (OLS) regression analysis is the basis for statistical process control. The effectiveness of monitoring a process is limited by attribute data. GLMs were applied to unite the linear and nonlinear regression fields and the model responses for different distributions. Myers and Montgomery, Nelder and Wedderburn have discussed GLM in detail. A GLM extrapolates the regression framework to wider scenarios in which a population does not require normality or the constant variance assumption, but instead requires three components: (1) exponential family distribution; (2) the function of the explanatory variable; and (3) the link function \( g(\mu_{ij}) \) given below

\[
g(\mu_{ij}) = X_j^T \beta_i = \eta_{ij}
\]  

where \( \mu_{ij} = E(Y_{ij}) \) denotes the expected value of the response variable \( Y_{ij} \) for laboratory \( i \); \( X_j \) is the explanatory variable vector that has terms \( X_{jk} \) \( (k = 1, \ldots, p) \), and \( \beta_i = [\beta_{i1}, \ldots, \beta_{ip}]^T \) is a \( p \times 1 \) parameter vector.

After these three components are specified, the GLM can be applied to generate a model that obtains estimates of measurement results and calculates the repeatability variance.

2.2. Procedure for repeatability calculation

As the square root of the average within-laboratory variance is the repeatability standard deviation, a GLM is utilized to acquire regression parameters that can calculate repeatability variance.

These procedures for calculating repeatability are described by IWLS, deviance analysis and repeatability variance. Section 2.2.1 presents an IWLS method for estimating regression parameters of a GLM, and Section 2.2.2 derives deviance statistics for performing goodness-of-fit tests for regression parameters. Section 2.2.3 presents a novel equation for calculating repeatability variance.

2.2.1. Iterative weighted least squares (IWLS)

The iterative equation employed to estimate parameter vector \( \beta_i \) is

\[
\beta_i^{(n)} = \beta_i^{(n-1)} + [\Sigma_i^{(n-1)}]^{-1} U_i^{(n-1)}
\]  

until \( \beta_i^{(n)} \) converges, superscript \( (n) \) denotes the \( n \)th iteration. The information matrix is \( U_i \) and \( \Sigma_i \) is the variance–covariance matrix, which are introduced in Equations (4) and (5), respectively.

Based on Equation (2), Equation (4) is utilized to obtain the maximum likelihood estimator (MLE) for parameters \( \beta_i \)

\[
U_{ik} = \frac{\partial L(\beta_i; Y_{ij})}{\partial \beta_{ik}} = \sum_{j=1}^{s} \frac{(Y_{ij} - \mu_{ij})}{\sigma_{ij}^2} \left( \frac{\partial \mu_{ij}}{\partial \eta_{ij}} \right) X_{jk}
\]  

where \( L(\beta_i; Y_{ij}) \) is the log-likelihood function for all the random variables \( Y_{ij} \), \( \sigma_{ij}^2 \) is the variance of \( Y_{ij} \), \( \eta_{ij} \) is the response variable of a link function and \( U_{ik} \) is the element of the variance–covariance matrix \( U_i \). This variance–covariance matrix \( U_i \) can be applied to derive the information matrix \( \Sigma_i \), which has terms \( \Sigma_{ilk} \)

\[
\Sigma_{ilk} = E[U_{il}U_{ik}] = \sum_{j=1}^{s} \frac{X_{jl}X_{jk}}{\sigma_{ij}^2} \left( \frac{\partial \mu_{ij}}{\partial \eta_{ij}} \right)^2
\]  

where \( l = 1, \ldots, p \).

This procedure is known as the IWLS method; see Charnes et al. for further details.
2.2.2. Deviance analysis

Deviance is a measure in the goodness-of-fit test for regression parameters. Theoretically, by utilizing deviance, whether a fitted model is significantly worse than a saturated model can be determined. To test null hypotheses and alternative hypotheses about parameter vector $\beta_i$ of length $p$ (Equation (6)), this study employs the Wald statistic as in Equation (7) and deviance statistics—also called the log-likelihood statistic in Equation (8)—to test the parameter estimates

$$H_0 : \beta_i = \beta_{i0} = \begin{bmatrix} \beta_{i1} \\ \vdots \\ \beta_{iq} \end{bmatrix}, \quad H_1 : \beta_i = \beta_{i1} = \begin{bmatrix} \beta_{i1} \\ \vdots \\ \beta_{ip} \end{bmatrix}$$

where $\beta_{i1}, \ldots, \beta_{iq}, \ldots, \beta_{ip}$ are parameters for $q < p$.

$$\left(\hat{\beta}_i - \beta_i\right)^T \left(\hat{\beta}_i - \beta_i\right) \sim \chi^2(p)$$

$$D = 2[L(\hat{\beta}_{i\text{max}}; Y_{ij}) - L(\hat{\beta}_i; Y_{ij})]$$

where $\hat{\beta}_i$ is the MLE of parameter vector $\beta_i$, and $\hat{\beta}_{i\text{max}}$ is the MLE of parameters in the saturated model.

Differences between deviance statistics in Equation (9) are utilized to test $H_0$ against $H_1$

$$\Delta D = D_0 - D_1 = 2[L(\hat{\beta}_{i\text{max}}; Y_{ij}) - L(\hat{\beta}_{i0}; Y_{ij})] \sim \chi^2(p - q)$$

If the value of $\Delta D$ is in the critical region, then $H_0$ is rejected and, thus, the hypotheses $H_1$ provides a significantly better description of the data; see Myers and Montgomery$^{26}$ and Dobson$^{29}$.

2.2.3. Construction of repeatability function

Estimated repeatability variance is defined by ASTM E 691$^{13}$ as average within-laboratory variance, which is the variation due to error as given below

$$\hat{\sigma}^2_{ri} = \sum_{j=1}^{s} (Y_{ij} - \hat{Y}_{ij})^2 / (s - p)$$

where $\hat{\sigma}^2_{ri}$ is the estimated variance of repeatability for laboratory $i$ and $\hat{Y}_{ij} = X_j^T \hat{\beta}_i$.

2.3. Procedure for reproducibility calculation

Reproducibility addresses variability between single test results obtained by different laboratories. Each laboratory applied the test method to examine specimens obtained randomly from a specific quantity of homogeneous material. The variation of reproducibility is typically high when an inspection is performed manually—human inspectors usually evaluate based on their experience. The variance of reproducibility can be obtained by the between-laboratory variance of laboratory means and the within-laboratory variance. Between-laboratory variance is discussed in Section 2.3.1 and reproducibility function is presented in Section 2.3.2.

2.3.1. Between-laboratory variance

Variation exists in the laboratory component of bias in laboratory $i$, which is called between-laboratory variance and represented as $VL$. This $VL$ is caused by the variance within operators and gauges; this variance includes random and system errors for different environments, gauge variation and inspection capability of an operator.

This study proposes the following process using three procedures to estimate $VL$ with a reasonable starting value. (This process is proved in Appendix A.)
Procedure 1 calculates $\Delta$

$$\Delta = \sum_{i=1}^{m} \left[ (VL + \hat{\sigma}^2_{Ri})^{-1} \times \bar{Y}_{i*} - \frac{\sum_{i=1}^{m} \sqrt{VL + \hat{\sigma}^2_{Ri}} \times \bar{Y}_{i*}}{m} \right]^2 - (m - 1)$$  \hspace{1cm} (11)

where $\hat{\sigma}^2_{Ri}$ is the variance of repeatability for laboratory $i$ (Equation (10)) and $\bar{Y}_{i*} = \sum_{j=1}^{s} Y_{ij}/s$. In line with the Mandel’s research $^{20}$, $10^{-8}$ can be used as a starting value for $VL$.

Procedure 2 calculates $\Gamma$ as

$$\Gamma = -2\sum_{i=1}^{m} \left[ \left( \sqrt{VL + \hat{\sigma}^2_{Ri}} \right)^{-1} \times \bar{Y}_{i*} - \frac{\sum_{i=1}^{m} \sqrt{VL + \hat{\sigma}^2_{Ri}} \times \bar{Y}_{i*}}{m} \right] \times \left[ \frac{\bar{Y}_{i*}}{2\sqrt{VL + \hat{\sigma}^2_{Ri}}} - \frac{1}{m} \times \sum_{i=1}^{m} \frac{\bar{Y}_{i*}}{2\sqrt{VL + \hat{\sigma}^2_{Ri}}} \right]$$  \hspace{1cm} (12)

Procedure 3 calculates a new $VL$

$$\text{New } VL = VL + \Delta/\Gamma$$  \hspace{1cm} (13)

These processes continue by substituting a new $VL$ for the previous value of $VL$. Iterations are then performed until $|\Delta/\Gamma|$ is sufficiently small, and the variance between laboratories is the convergent value of $VL$, which is represented as $\hat{\sigma}^2_{L}$.

### 2.3.2. Construction of the reproducibility function

The variance among individual test results obtained in different laboratories is the sum of the within-laboratory variance and between-laboratory variance of laboratory means. Thus, the reproducibility variance is defined by ASTM E 691 $^{13}$ and Mandel $^{30}$ as follows:

$$\hat{\sigma}^2_{Ri} = \hat{\sigma}^2_{L} + \hat{\sigma}^2_{Ri}$$  \hspace{1cm} (14)

where $\hat{\sigma}^2_{Ri}$ signifies the reproducibility variance for laboratory $i$ and $\hat{\sigma}^2_{L}$ is the variance between laboratories.

Based on this definition, Mandel $^{20}$ could evaluate the repeatability variance and the reproducibility variance for any measurement, whether continuous or discrete.

### 2.4. Evaluation of gauge capability

Equation (15) is used to evaluate the data to determine if test equipment is suited for its intended purpose

$$\% R&R = 100 \times \sqrt{\frac{\hat{\sigma}^2_{Ri} + \hat{\sigma}^2_{Ri}}{\sigma_i}}$$  \hspace{1cm} (15)

where $\sigma_i$ is the standard deviation of the process.

The criteria for acceptability are dependent on the percentage of part tolerance that is consumed by test equipment error. Acceptability is based on the following criteria, which are approved by ASTM F 1469 $^{31}$:

1. Test equipment with an R&R percentage 10% or less is fully capable and can certify tests.
2. Test equipment with an R&R percentage of 10–30% shall be reviewed by a qualified technician. If possible, the test system should be improved or replaced; however, this system can be used as is until an improvement is found.
3. Test equipment with an R&R percentage $>30\%$ is unacceptable. Immediate corrective action should be taken to replace or improve the test system.

### 2.5. Model for gauge R&R

To summarize these discussions, this study presents a novel model to evaluate a gauge R&R containing six modules: IWLS; deviance analysis; repeatability variance; between-laboratory variance; reproducibility
variance; and gauge capability evaluation. Figure 1 presents the flowchart of the proposed model. IWLS was applied to obtain the MLE of regression parameters. Deviance analysis was utilized to assess the significance of regression parameters. Repeatability variance was used to derive the estimator of variance for repeatability. Between-laboratory variance was used to acquire the estimator of variance between laboratories. Reproducibility variance was employed to acquire the estimator of variance for reproducibility. Gauge capability evaluation was applied to examine gauge capability.

3. **AN EXAMPLE**

The case company is a specialty steel company that manufactures special alloy materials. Non-metallic inclusion in melting, forging, and rolling accounts for numerous product defects. Contamination by non-metallic materials in wrought steel can cause brittle machining, reduced strength and poor ductility. During the rolling process, such contamination can result in cracking and ruin the glossy surface on castings. To test for non-metallic inclusions, the variation of measurement results is rooted in ergonomics, causing an inspector to make an incorrect decision. Such decisions result in producer and consumer risk. In this example, the proposed model was applied to examine gauge capability in inspecting for non-metallic inclusions in six laboratories.
3.1. Design of experiment

The standard test method (ASTM E 4532) was used to determine the non-metallic inclusion content in alloy steel by metallographic photomicrography. The following experiment demonstrates how this model is applied. The specimen was surveyed at 100 ×; the polished surface area was 160 mm² (9.5 mm × 19 mm). Every 0.50 mm² field on the polished surface was examined for inclusion Type D and compared with the square fields on Plate 1–r by the jernkontoret (JK) inclusion rating—a method of measuring non-metallic inclusions based on Swedish Jernkontoret procedures. Appraisers prepared the specimens and examined each specimen with an electron microscope (ASTM E 333). Spots on the sample surfaces were inclusions. In this context, a lot is defined as a unit of material processed at one time and subjected to processing variables similar to other lots. For cross-section thickness <0.95 mm, 10 longitudinal surface pieces from each sampling location were mounted as suitable specimens for polishing. To generate a reasonable estimate of inclusion variations within a lot, at least six locations were chosen as representative as possible of the lot and were examined repeatedly. To estimate the reproducibility variance, 10 samples were inspected by six laboratories. For a good estimate of inclusion, each specimen was microscopically examined at the six locations and the smallest inclusion number at these six locations is the representative value.

3.2. Gauge R&R model results

This study applied the gauge R&R model to evaluate the gauge capability for six laboratories. Section 3.2.1 demonstrates an application of the proposed model. In Section 3.2.2, the experimental result is used to conduct a gauge capability study.

3.2.1. Application of gauge R&R model

Table I presents the application of the gauge R&R model in the case. The IWLS procedure was employed to obtain the MLEs of Poisson regression parameters for a GLM. Deviance statistics for the Poisson model were applied to fit experimental data to Poisson regression parameters. Repeatability variance was used to calculate the variance of repeatability for six laboratories. Between-laboratory variance was employed to generate the estimator of variance between six laboratories. Reproducibility variance calculated the variance of reproducibility for six laboratories.

3.2.2. Experimental results

The IWLS procedure was used to derive Poisson regression parameters (module 1 in Table I). The responses Y_ij are the number of inclusions for specimen j in laboratory i, which were defects in steel identified by metallographic photomicrography. The X_j2 are longitudinal surface pieces from the sampling specimens. For the defects belonging to Poisson random variables, the expected values are equal to the variance and are represented as Procedure 1.1. Procedure 1.2 is the process of IWLS that continues until \( \hat{\beta}_1^{(n)} \) converges at \( \hat{\beta}_2^{(n)} \). This study used a simple linear regression to obtain an initial value \( \hat{\beta}_1^{(0)} \) (column 4 in Table II). Column 5 in Table II presents the convergent values for the six laboratories. The significance level of the Poisson regression parameter in laboratory i was tested (module 2 in Table I). The null hypotheses and alternative hypotheses are stated as Procedure 2.1 and deviance statistics for this Poisson model are calculated by Procedure 2.2. Column 6 in Table II presents the values of deviance, which were all less than the lower 5% tail of the distribution (\( \chi^2_{0.05}(8) = 15.5073 \)), indicating that the entire model fits the data well.

The variance of repeatability for laboratory i (column 2 in Table III) was calculated by Procedure 3.1 (module 3 in Table I). The variance between laboratories was calculated by Procedures 4.1, 4.2, and 4.3 (module 4 in Table I) and the variance between laboratories was 1.6035. The variance of reproducibility (column 3 in Table III) was derived by Procedure 5.1 (module 5 in Table I). The formula of evaluating gauge capability (module 6 in Table I) was calculated by Procedure 6.1.

Based on column 4 in Table III, Laboratory 1 has the highest percentage R&R with a value over 30% and, thus, the gauge capability was unacceptable. Laboratory 1 must take immediate corrective action to replace
Table I. Detailed procedure to calculate gauge R&R

<table>
<thead>
<tr>
<th>Module</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( E(Y_{ij}) = \text{Var}(Y_{ij}) = \hat{\beta}<em>{i1} + \hat{\beta}</em>{i2}X_{j2}, \quad i = 1, \ldots, 6, \quad j = 1, \ldots, 10 )</td>
</tr>
<tr>
<td>1.2</td>
<td>[ \sum_{j=1}^{10} \frac{1}{\hat{\beta}<em>{i1} + \hat{\beta}</em>{i2}X_{j2}} \sum_{j=1}^{10} X_{j2} \left( \frac{1}{\hat{\beta}<em>{i1} + \hat{\beta}</em>{i2}X_{j2}} \right)^{(n-1)} ]</td>
</tr>
<tr>
<td>2</td>
<td>( H_0: \beta_i = \beta_{i0} = \begin{bmatrix} \hat{\beta}<em>{i1} \ \hat{\beta}</em>{i2} \end{bmatrix}, \quad H_1: \beta_i \neq \beta_{i0} )</td>
</tr>
<tr>
<td>2.2</td>
<td>Deviance = ( 2 \sum_{j=1}^{10} [Y_{ij}(\log Y_{ij} - \log \hat{Y}<em>{ij}) - (Y</em>{ij} - \hat{Y}_{ij})] )</td>
</tr>
<tr>
<td>3</td>
<td>( \hat{\sigma}<em>{ri}^2 = \sum</em>{j=1}^{10} (Y_{ij} - \hat{Y}_{ij})^2 / 8 )</td>
</tr>
<tr>
<td>4</td>
<td>( \Delta = 6 \sum_{i=1}^{10} \left[ \sqrt{(VL + \hat{\sigma}<em>{ri}^2)}^{-1} \times \overline{Y}</em>{ri} - \frac{\sum_{i=1}^{6} \sqrt{(VL + \hat{\sigma}<em>{ri}^2)}^{-1}}{6} \times \overline{Y}</em>{ri} \right]^2 )</td>
</tr>
<tr>
<td>4.2</td>
<td>( \Gamma = -2 \sum_{i=1}^{10} \left[ \sqrt{(VL + \hat{\sigma}<em>{ri}^2)}^{-1} \times \overline{Y}</em>{ri} - \frac{\sum_{i=1}^{6} \sqrt{(VL + \hat{\sigma}<em>{ri}^2)}^{-1}}{6} \times \overline{Y}</em>{ri} \right] \times \left[ 2 \sqrt{(VL + \hat{\sigma}<em>{ri}^2)}^{-1} - \frac{1}{6} \times \sum</em>{i=1}^{6} \frac{\overline{Y}<em>{ri}}{2 \sqrt{(VL + \hat{\sigma}</em>{ri}^2)}} \right] \times (VL + \hat{\sigma}_{ri}^2)^{-2} )</td>
</tr>
<tr>
<td>4.3</td>
<td>New ( VL = VL + \Delta / \Gamma )</td>
</tr>
<tr>
<td>5</td>
<td>( \hat{\sigma}<em>{RI}^2 = VL + \hat{\sigma}</em>{ri}^2, \quad i = 1, \ldots, 6 )</td>
</tr>
<tr>
<td>6</td>
<td>( % \ R&amp;R = 100 \times \sqrt{\hat{\sigma}<em>{ri}^2 + \hat{\sigma}</em>{RI}^2} / \sigma_t )</td>
</tr>
</tbody>
</table>

Table II. Deviance analysis in case company

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Laboratory</th>
<th>Mean</th>
<th>Variance</th>
<th>((\hat{\beta}<em>{i1}^{(0)}, \hat{\beta}</em>{i2}^{(0)}))</th>
<th>((\hat{\beta}<em>{i1}^{(n)}, \hat{\beta}</em>{i2}^{(n)}))</th>
<th>Deviance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>3.3</td>
<td>5.5668</td>
<td>(3.3, -0.0364)</td>
<td>(3.2999, -0.0430)</td>
<td>13.2039</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.1</td>
<td>4.5446</td>
<td>(3.1, 0.0273)</td>
<td>(3.0999, 0.03355)</td>
<td>11.7774</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.7</td>
<td>2.6787</td>
<td>(3.7, -0.1455)</td>
<td>(3.6999, -0.1670)</td>
<td>5.2939</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.3</td>
<td>2.6787</td>
<td>(3.3, -0.0546)</td>
<td>(3.2999, -0.0598)</td>
<td>7.0965</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>3.5</td>
<td>2.9443</td>
<td>(3.5, -0.1364)</td>
<td>(3.4999, -0.1565)</td>
<td>5.5853</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3.5</td>
<td>3.1666</td>
<td>(3.5, -0.2273)</td>
<td>(3.4999, -0.2633)</td>
<td>5.0093</td>
</tr>
</tbody>
</table>

or improve the test system. The percentage R&R of Laboratory 2 is above 30% and the gauge capability requires improvement. To improve measurement quality, Laboratory 2 must take action to enhance the test system. Laboratories 3–6 had percentage R&Rs of 20–30%; thus, gauge capability was marginal and requires monitoring. These laboratories must be reviewed by a qualified technician. The system can continue to be utilized until an improvement is introduced. High reproducibility variance may be due to inspectors.
Table III. Results of repeatability variance and reproducibility variance

<table>
<thead>
<tr>
<th>Laboratory</th>
<th>Repeatability variance</th>
<th>Reproducibility variance</th>
<th>% R&amp;R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7026</td>
<td>3.0086</td>
<td>31.52</td>
</tr>
<tr>
<td>2</td>
<td>0.5741</td>
<td>2.7517</td>
<td>30.88</td>
</tr>
<tr>
<td>3</td>
<td>0.3069</td>
<td>2.2173</td>
<td>27.50</td>
</tr>
<tr>
<td>4</td>
<td>0.3344</td>
<td>2.2722</td>
<td>27.86</td>
</tr>
<tr>
<td>5</td>
<td>0.3446</td>
<td>2.2927</td>
<td>28.00</td>
</tr>
<tr>
<td>6</td>
<td>0.3229</td>
<td>2.2493</td>
<td>27.71</td>
</tr>
</tbody>
</table>

with insufficient experience, poor inspection culture, improper training, etc. High repeatability variance can result from improper calibration, maintenance, measurement uncertainty, etc. The improvements of these poor measurement systems can be achieved by the following method. Appraisers require increased operation and discrimination training for using measurement equipment. The equipment must be calibrated and the resolution scale should be enhanced.

The proposed model can identify the optimal gauge capability from numerous inspection systems, reduce producer risk (Type I error) and consumer risk (Type II error) by enhancing measurement capability and improving manufacturing processes to obtain the objective of the six sigma methods of 3.4 parts per million (ppm).

4. CONCLUSIONS

This research presented a novel method for calculating the gauge capability when inspecting attribute data that can estimate R&R variances. These approaches are alternatives to the quantitative measurements in ASTM E 691 or ISO/TS 16949. Alternative estimation procedures were used in a GLM and iterative process methods. This study reviewed the procedures of gauge capability for pass–fail inspection, demonstrated the use of IWLS to generate MLEs of parameters and deviance analysis to verify that the model fits the data well, and integrated these approaches to derive a point estimate of repeatability variance. To determine reproducibility variation, the proposed scheme used an iterative process to obtain variation for laboratory components of bias and used ASTM’s definition to acquire the variance of reproducibility. This study applied this alternative method to a case to calculate the estimator of R&R.

The proposed model can be employed in inspecting contamination inspection, and in inspecting defective pixels on a TFT-LCD panel, defects of a PCB, defects on an integrated circuit (IC) wafer, etc. The proposed model can be applied to calculate R&R for attribute data.

The ASTM E 691 focuses on calculating R&R of variable data. The ISO/TS 16949 standard can verify an inspector’s determination for go/no go gauge only. This proposed method can supplement the shortcomings of ASTM E 691 and ISO/TS 16949.

The distribution of attribute data must be considered when conducting a gauge capability study. Such considerations include the tendency of defects to cluster in a region in an inspecting unit, and that the conventional Poisson distribution is insufficient for modeling defect clustering. Integrating compound or mixed Poisson statistics into estimates of gauge R&R using a GLM is a research direction for future.

REFERENCES


APPENDIX A

The variance among individual measurement results obtained in different laboratories is the sum of the within-laboratory and the between-laboratory variances of laboratory means. Thus, the reproducibility variance is the sum of between-laboratory variance and the repeatability variance, and the repeatability variance can be estimated as the mean square error within a laboratory by GLM. Between-laboratory variance must be calculated to estimate reproducibility variance.

Let $Y_{i*}$ be the random variable of measurement result for laboratory $i$

$$Y_{i*} = \mu + \text{Lab}_i + e_i$$  \hspace{1cm} (A1)

where $\mu$ is the true average value of measurement results in all laboratories, $\text{Lab}_i$ is the amount by which laboratory $i$ deviates from this true value, and $e_i$ is the random within-laboratory error of each measurement $Y_{i*}$.

The variation of $Y_{i*}$ is

$$\text{Var}(Y_{i*}) = \text{Var}(\text{Lab}_i) + \text{Var}(e_i)$$  \hspace{1cm} (A2)

$VL$ is assumed to be the best estimator of $\text{Var}(\text{Lab}_i)$, $\text{Var}(e_i)$ can be estimated by GLM as (Equation (10)) and represented as $\hat{\sigma}_{ri}^2$, then (A2) can be represented as

$$\hat{\text{Var}}(Y_{i*}) = \frac{\hat{\text{Var}}(Y_{i*})}{VL + \hat{\sigma}_{ri}^2} = 1$$  \hspace{1cm} (A3)

$$\hat{\text{Var}}(\sqrt{(VL + \hat{\sigma}_{ri}^2)^{-1} \times Y_{i*}}) = 1$$  \hspace{1cm} (A4)

An estimator of $\hat{\text{Var}}(\sqrt{(VL + \hat{\sigma}_{ri}^2)^{-1} \times Y_{i*}})$ is given by

$$\hat{\text{Var}}(\sqrt{(VL + \hat{\sigma}_{ri}^2)^{-1} \times Y_{i*}}) = \frac{\sum_{i=1}^{m} \left[ \sqrt{(VL + \hat{\sigma}_{ri}^2)^{-1} \times Y_{i*}} - \frac{\sum_{i=1}^{m} \sqrt{(VL + \hat{\sigma}_{ri}^2)^{-1} \times Y_{i*}}}{m} \right]^2}{m - 1}$$  \hspace{1cm} (A5)

$$\Delta = \sum_{i=1}^{m} \left[ \sqrt{(VL + \hat{\sigma}_{ri}^2)^{-1} \times Y_{i*}} - \frac{\sum_{i=1}^{m} \sqrt{(VL + \hat{\sigma}_{ri}^2)^{-1} \times Y_{i*}}}{m} \right]^2 - (m - 1)$$  \hspace{1cm} (A6)
When \( \Delta \) is 0, the best estimator of \( V(Lab_i) \) can be obtained. Since \( \Delta \) depends on \( VL \), using derivative by chain rule and the iterative process can estimate this value

\[
\Gamma = \frac{\partial \Delta}{\partial VL} = -2 \sum_{i=1}^{m} \left[ \sqrt{(VL + \hat{\sigma}_r^2)^{-1}} \times \bar{Y}_{i^*} - \frac{1}{m} \sum_{i=1}^{m} \frac{\bar{Y}_{i^*}}{\sqrt{(VL + \hat{\sigma}_r^2)^{-1}}} \right] \\
\times \left[ \frac{\bar{Y}_{i^*}}{\sqrt{(VL + \hat{\sigma}_r^2)^{-1}}} - \frac{1}{m} \sum_{i=1}^{m} \frac{\bar{Y}_{i^*}}{\sqrt{(VL + \hat{\sigma}_r^2)^{-1}}} \right] \times (VL + \hat{\sigma}_r^2)^{-2} \tag{A7}
\]

\[
d\Delta = \frac{\partial \Delta}{\partial VL} \, dVL, \quad dVL = \frac{d\Delta}{\partial VL} = \frac{0 - \Delta}{\Gamma} \tag{A8}
\]

Apply the following algorithm to acquire \( VL \). First, initializing value \( VL^{(0)} = 10^{-8} \) and \( \Delta^{(0)} \), \( \Gamma^{(0)} \) can be obtained by (A6) and (A7).

If \( \Delta^{(n)} \) is not 0

\[
VL^{(n)} = VL^{(n-1)} + dVL = VL^{(n-1)} + \frac{0 - \Delta^{(n-1)}}{\Gamma^{(n-1)}} \tag{A9}
\]

Substitute \( VL^{(n)} \) into (A6), (A7) and iterate this process until \( \Delta^{(n)} = 0 \), where subscript \((n)\) represents the \( n \)th iteration. Finally, the best estimator of \( Var(Lab_i) \) can be obtained.

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