國立雲林科技大學 工業工程與管理所
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品質與可靠度工程實驗室
Quality and Reliability Engineering Lab.

全因子設計

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目前：http://140.125.88.116/QRE

大綱

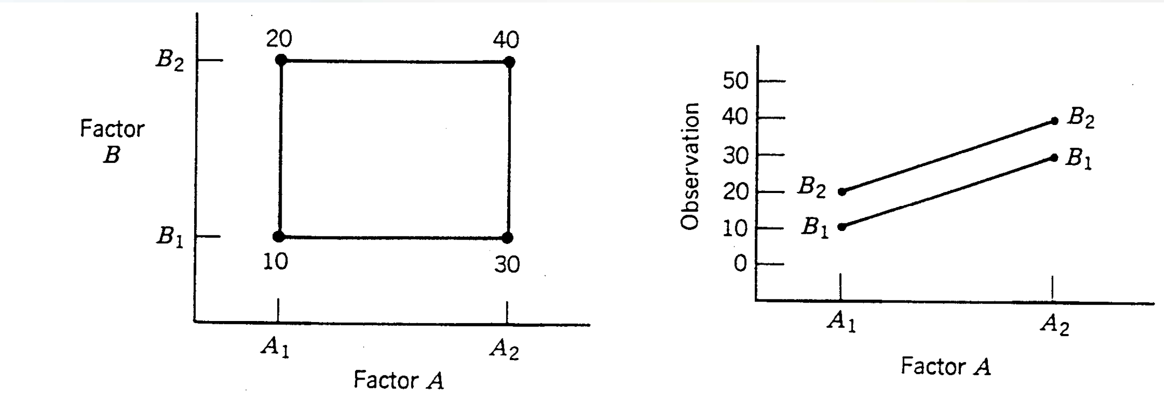
1. 全因子設計
2. 二因子實驗設計
3. 三因子實驗設計
4. 2k因子階層設計

1. 全因子設計

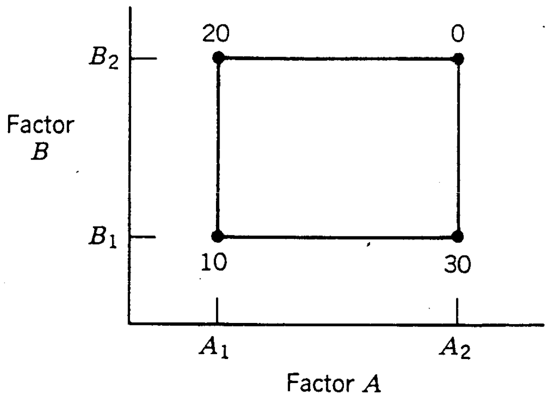
- 實驗設計是一種用於改善製程的重要工具。
- 對於新製程的開發可以降低變異，使產品與目標一致， 減少開發時間。
- 對於新產品的開發可以決定影響結果的關鍵設計參數。

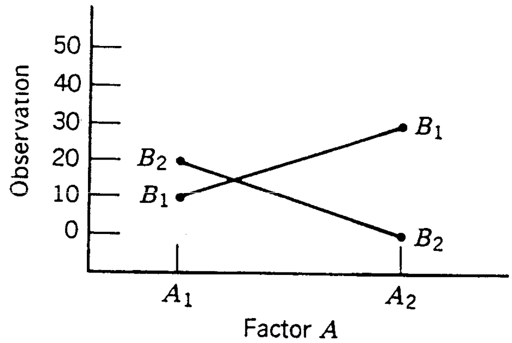
2. 二因子實驗設計

- 二因子無交互作用

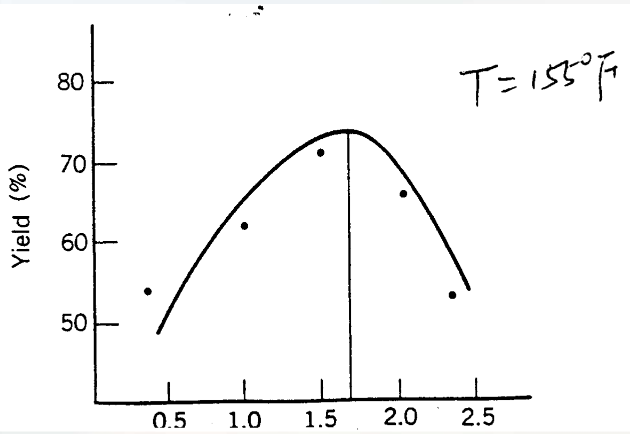


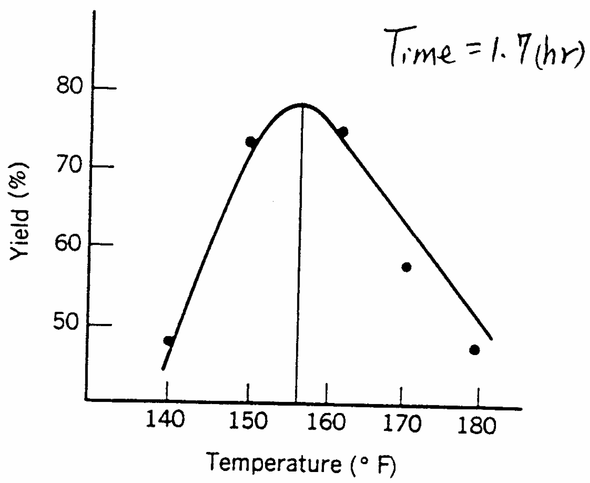
- 二因子有交互作用

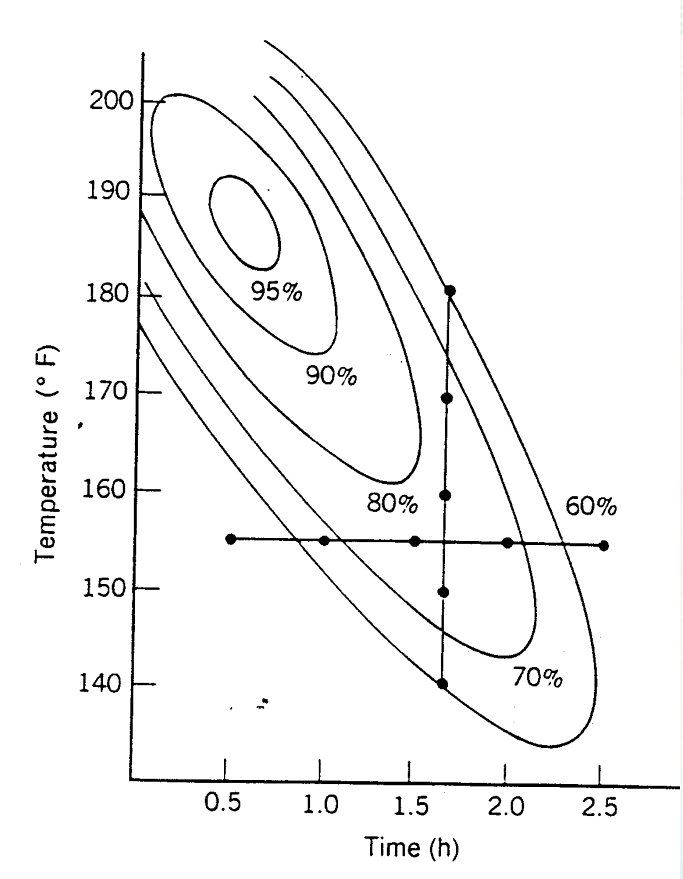




One-factor at a time 之方法







二因子實驗設計之模式

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \\ k = 1, 2, \ldots, n \end{cases}$$

HypothesisTesting :

$$1. \begin{cases} H_0 : \tau_1 = \tau_2 = \cdots = \tau_a = 0 \\ H_1 : \text{at least one } \tau_i \neq 0 \end{cases}$$

$$2. \begin{cases} H_0 : \beta_1 = \beta_2 = \cdots = \beta_b = 0 \\ H_1 : \text{at least one } \beta_j \neq 0 \end{cases}$$

$$3. \begin{cases} H_0 : (\tau\beta)_{ij} = 0, \text{ for all } i, j \\ H_1 : \text{at least one } (\tau\beta)_{ij} \neq 0 \end{cases}$$

Data Sheet

Data for a two-factor factorial design

		Factor B				
		1	2 $\quad y_{ij\cdot}$	$\cdots$	b	$y_{i\cdot\cdot}$
Factor A	1	$y_{111}, y_{112}, \ldots, y_{11n}$	$y_{121}, y_{122}, \ldots, y_{12n}$		$y_{1b1}, y_{1b2}, \ldots, y_{1bn}$	$y_{1\cdot\cdot}$
	2	$y_{211}, y_{212}, \ldots, y_{21n}$	$y_{221}, y_{222}, \ldots, y_{22n}$		$y_{2b1}, y_{2b2}, \ldots, y_{2bn}$	$y_{2\cdot\cdot}$
	$\vdots$					
	a	$y_{a11}, y_{a12}, \ldots, y_{a1n}$	$y_{a21}, y_{a22}, \ldots, y_{a2n}$		$y_{ab1}, y_{ab2}, \ldots, y_{abn}$	$y_{a\cdot\cdot}$
	$y_{\cdot j\cdot}$					

ANOVA表 – Two-Factor Factorial

The analysis of variance table for a two-factor factorial, fixed effects model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A	SS_A	$a-1$	$MS_A = \dfrac{SS_A}{a-1}$	$F_0 = \dfrac{MS_A}{MS_E}$
B	SS_B	$b-1$	$MS_B = \dfrac{SS_B}{b-1}$	$F_0 = \dfrac{MS_B}{MS_E}$
Interaction	SS_{AB}	$(a-1)(b-1)$	$MS_{AB} = \dfrac{SS_{AB}}{(a-1)(b-1)}$	$F_0 = \dfrac{MS_{AB}}{MS_E}$
Error	SS_E	$ab(n-1)$	$MS_E = \dfrac{SS_E}{ab(n-1)}$	
Total	SS_T	$abn-1$		

其中

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_B = \sum_{j=1}^{b} \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$$

$$SS_E = SS_T - SS_{\text{subtotals}}$$

$$SS_{AB} = SS_{\text{subtotals}} - SS_A - SS_B$$

$$SS_{subtotals} = \sum_{i=1}^{a}\sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn}$$

Example

Adhesion force data

	Application Method		
Primer Type	Dipping	Spraying	$y_{i..}$
1	4.0, 4.5, 4.3 12.8	5.4, 4.9, 5.6 15.9	28.7
2	5.6, 4.9, 5.4 15.9	5.8, 6.1, 6.3 18.2	34.1
3	3.8, 3.7, 4.0 11.5	5.5, 5.0, 5.0 15.5	27.0
$y_{.j.}$	40.2	49.6	$89.8 = y_{...}$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$= (4.0)^2 + (4.5)^2 + \cdots + (5.0)^2 - \frac{(89.8)^2}{18} = 10.72$$

$$SS_{\text{primers}} = \sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$= \frac{(28.7)^2 + (34.1)^2 + (27.0)^2}{6} - \frac{(89.8)^2}{18} = 4.58$$

$$SS_{\text{methods}} = \sum_{j=1}^{b} \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$$

$$= \frac{(40.2)^2 + (49.6)^2}{9} - \frac{(89.8)^2}{18} = 4.91$$

$$SS_{\text{interaction}} = \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn} - SS_{\text{primers}} - SS_{\text{methods}}$$

$$= \frac{(12.8)^2 + (15.9)^2 + (11.5)^2 + (15.9)^2 + (18.2)^2 + (15.5)^2}{3}$$

$$- \frac{(89.8)^2}{18} - 4.58 - 4.91 = 0.24$$

$$SS_E = SS_T - SS_{\text{primers}} - SS_{\text{method}} - SS_{\text{interaction}}$$

$$= 10.72 - 4.58 - 4.91 - 0.24 = 0.99$$

Analysis of variance for Example

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Primer types	4.58	2	2.29	28.63
Application methods	4.91	1	4.91	61.38
Interaction	0.24	2	0.12	1.5
Error	0.99	12	0.08	
Total	10.72	17		

- 決策模式：
  - 因為F_0(Primer Types) = 28.63 > $F_{0.05,2,12}$ = 3.89

    F_0(Application Methods) = 61.38 > $F_{0.05,1,12}$ = 4.75

    所以此二因子對黏著力皆有顯著影響。
  - 但 F_0(Interaction) = 1.5 < $F_{0.05,2,12}$ = 3.89，所以此二因子的交互作用對黏著力無明顯之影響。

Another Example

Table 5-1 Life (in hours) Data for the Battery Design Example

Material Type	Temperature (°F) 15		70		125	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

ANOVA Results

Table 5-5 Analysis of Variance for Battery Life Data

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Material types	10,683.72	2	5,341.86	7.91	0.0020
Temperature	39,118.72	2	19,559.36	28.97	0.0001
Interaction	9,613.78	4	2,403.44	3.56	0.0186
Error	18,230.75	27	675.21		
Total	77,646.97	35			

- Computer Output (Model Adequacy Checking)

Multiple Comparisons

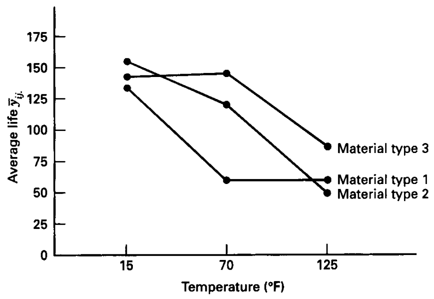


Figure 5-9 Material type–temperature plot for Example 5-1.

$$\bar{y}_{12.} = 57.25 \quad \text{(material type 1)}$$
$$\bar{y}_{22.} = 119.75 \quad \text{(material type 2)}$$
$$\bar{y}_{32.} = 145.75 \quad \text{(material type 3)}$$

and

$$T_{0.05} = q_{0.05}(3, 27)\sqrt{\frac{MS_E}{n}} = 3.50\sqrt{\frac{675.21}{4}} = 45.47$$

where we obtained $q_{0.05}(3, 27) \simeq 3.50$ by interpolation in Appendix Table VIII. The pairwise comparisons yield

3 vs. 1: $145.75 - 57.25 = 88.50 > T_{0.05} = 45.47$

3 vs. 2: $145.75 - 119.75 = 26.00 < T_{0.05} = 45.47$

2 vs. 1: $119.75 - 57.25 = 62.50 > T_{0.05} = 45.47$

3. 三因子實驗設計

$$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\beta\gamma)_{ijk} + \epsilon_{ijkl} \quad \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \\ k = 1, 2, \ldots, c \\ l = 1, 2, \ldots, n \end{cases}$$

The ANOVA TABLE

Table 5-12 The Analysis of Variance Table for the Three-Factor Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	Expected Mean Square	F_0
A	SS_A	$a-1$	MS_A	$\sigma + \frac{bcn \sum \tau_i^2}{a-1}$	$F_0 = \frac{MS_A}{MS_E}$
B	SS_B	$b-1$	MS_B	$\sigma^2 + \frac{acn \sum \beta_j^2}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
C	SS_C	$c-1$	MS_C	$\sigma^2 + \frac{abn \sum \gamma_k^2}{c-1}$	$F_0 = \frac{MS_C}{MS_E}$
AB	SS_{AB}	$(a-1)(b-1)$	MS_{AB}	$\sigma^2 + \frac{cn \sum \sum (\tau\beta)_{ij}^2}{(a-1)(b-1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
AC	SS_{AC}	$(a-1)(c-1)$	MS_{AC}	$\sigma^2 + \frac{bn \sum \sum (\tau\gamma)_{ik}^2}{(a-1)(c-1)}$	$F_0 = \frac{MS_{AC}}{MS_E}$
BC	SS_{BC}	$(b-1)(c-1)$	MS_{BC}	$\sigma^2 + \frac{an \sum \sum (\beta\gamma)_{jk}^2}{(b-1)(c-1)}$	$F_0 = \frac{MS_{BC}}{MS_E}$
ABC	SS_{ABC}	$(a-1)(b-1)(c-1)$	MS_{ABC}	$\sigma^2 + \frac{n \sum \sum \sum (\tau\beta\gamma)_{ijk}^2}{(a-1)(b-1)(c-1)}$	$F_0 = \frac{MS_{ABC}}{MS_E}$
Error	SS_E	$abc(n-1)$	MS_E	σ^2	
Total	SS_T	$abcn-1$			

An Example

Table 5-13 Fill Height Deviation Data for Example 5-3

	Operating Pressure (B)				
	25 psi		30 psi		
	Line Speed (C)		Line Speed (C)		
Percent Carbonation (A)	200	250	200	250	$y_{i...}$
10	−3, −1 (−4)	−1, 0 (−1)	−1, 0 (−1)	1, 1 (2)	−4
12	0, 1 (1)	2, 1 (3)	2, 3 (5)	6, 5 (11)	20
14	5, 4 (9)	7, 6 (13)	7, 9 (16)	10, 11 (21)	59
$B \times C$ Totals $y_{.jk.}$	6	15	20	34	$75 = y_{....}$
$y_{.j..}$	21		54		

$A \times B$ Totals

$y_{ij..}$

A \ B	25	30
10	−5	1
12	4	16
14	22	37

$A \times C$ Totals

$y_{i.k.}$

A \ C	200	250
10	−5	1
12	6	14
14	25	34

The Results

Table 5-14 Analysis of Variance for Example 5-3

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Percentage of carbonation (A)	252.750	2	126.375	178.412	<0.0001
Operating pressure (B)	45.375	1	45.375	64.059	<0.0001
Line speed (C)	22.042	1	22.042	31.118	0.0001
AB	5.250	2	2.625	3.706	0.0558
AC	0.583	2	0.292	0.412	0.6713
BC	1.042	1	1.042	1.471	0.2485
ABC	1.083	2	0.542	0.765	0.4867
Error	8.500	12	0.708		
Total	336.625	23			

- Computer Results

4. 2^k因子階層設計

- k個因子，每個因子2個水準(+,-) ，共2^k次實驗（當 n = 1 時）。
- 在因子數不多的狀況下，常用於實驗初期，來了解因子對反應變數之可能影響。
- 只能看出因子對反應變數之線性作用(linear effect) ，無法預估高階曲面作用。

The 2^2 Factorial Design

Factor A	Factor B	Treatment Combination	Replicate I	Replicate II	Replicate III	Total
−	−	A low, B low	28	25	27	80
+	−	A high, B low	36	32	32	100
−	+	A low, B high	18	19	23	60
+	+	A high, B high	31	30	29	90

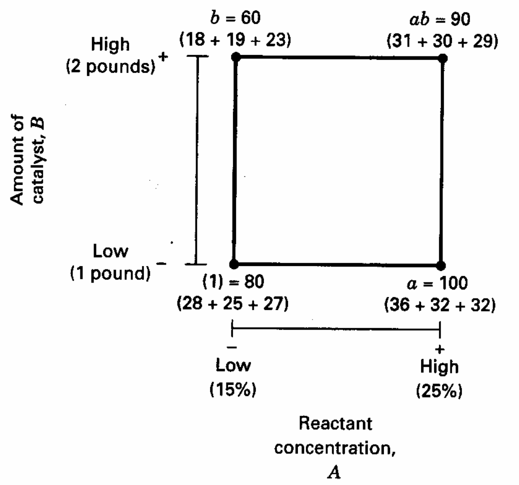


Figure 6-1 Treatment combinations in the 2^2 design.

The Calculation (I)

- Effects

$$A = \frac{1}{2n}\{[ab-b]+[a-(1)]\} = \frac{1}{2n}[ab+a-b-(1)]$$

$$B = \frac{1}{2n}\{[ab-a]+[b-(1)]\} = \frac{1}{2n}[ab+b-a-(1)]$$

$$AB = \frac{1}{2n}\{[ab-b]-[a-(1)]\} = \frac{1}{2n}[ab+(1)-a-b]$$

- Contrast

$$Contrast_A = ab+a-b-(1)$$

$$Contrast_B = ab+b-a-(1)$$

$$Contrast_{AB} = ab+(1)-a-b$$

$$=> A = \frac{Contrast_A}{n2^{k-1}}$$

The Calculation (II)

- Sum of Square Errors

$$SS_A = \frac{[ab + a - b - (1)]^2}{4n} = \frac{(Contrast_A)^2}{n2^k}$$

$$SS_B = \frac{(Contrast_B)^2}{n2^k}$$

$$SS_{AB} = \frac{(Contrast_{AB})^2}{n2^k}$$

$$SS_T = \sum\sum\sum\left(y_{ijk} - \bar{y}_{...}\right)^2 = \sum\sum\sum y_{ijk}^2 - \frac{y_{...}^2}{4n}$$

$$SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

The AVONA Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
A	208.33	1	208.33	53.15	0.0001
B	75.00	1	75.00	19.13	0.0024
AB	8.33	1	8.33	2.13	0.1826
Error	31.34	8	3.92		
Total	323.00	11			

Table 6-2 Algebraic Signs for Calculating Effects in the 2^2 Design

Treatment Combination	Factorial Effect *I*	*A*	*B*	*AB*
(1)	+	−	−	+
a	+	+	−	−
b	+	−	+	−
ab	+	+	+	+

The Regression Model

- Since the effect of AB is not significant, the regression model would be

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

  - The estimates of these coefficients are

$$\hat{\beta}_0 = \bar{y}_{...}$$
$$\hat{\beta}_1 = \frac{Effect_A}{2}$$
$$\hat{\beta}_2 = \frac{Effect_B}{2}$$

Why?

The Residuals

- Residuals at $x_1=1$ and $x_2 = -1$

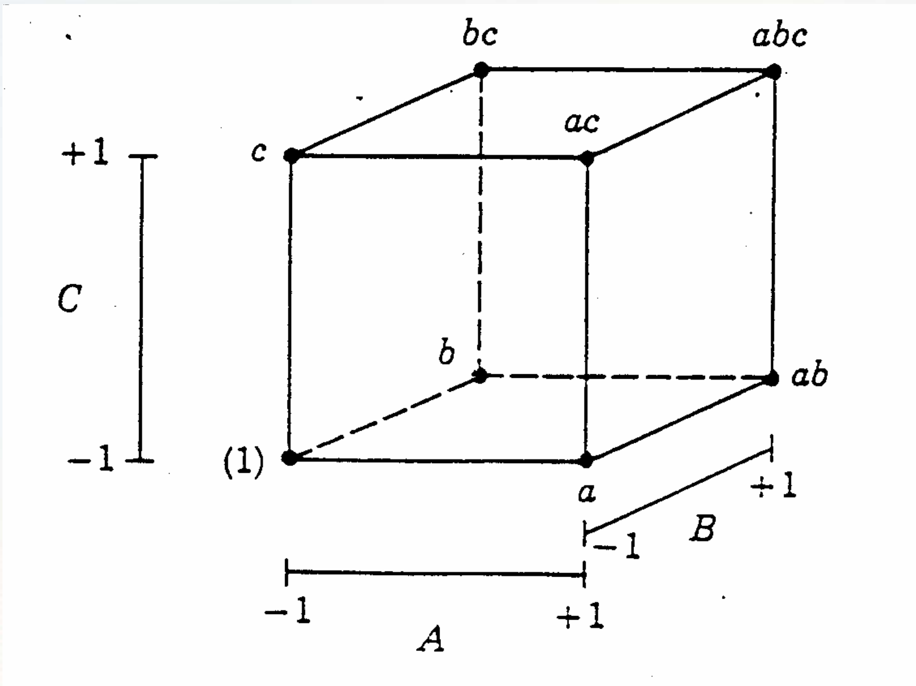
$$\hat{y} = 27.5 + \left(\frac{8.33}{2}\right)(+1) + \left(\frac{-5.00}{2}\right)(-1) = 34.165$$

and

$$e_4 = 36 - 34.165 = 1.835$$
$$e_5 = 32 - 34.165 = -2.165$$
$$e_6 = 32 - 34.165 = -2.165$$

  - Computer Output

2^3因子階層設計



2^3因子階層設計_符號表

Signs for effects in the 2^3 design

Treatment Combination	Factorial Effect							
	I	A	B	AB	C	AC	BC	ABC
(1)	+	−	−	+	−	+	+	−
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
ab	+	+	+	+	−	−	−	−
c	+	−	−	+	+	−	−	+
ac	+	+	−	−	+	+	−	−
bc	+	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+	+

計算

- 各因子之 Contrast = $\sum(Sign) \times (Treatment\ Combination)$
  - $Contrast_A$ = -(1)+a-b+ab-c+ac-bc+abc
  - $Contrast_B$ = -(1)-a+b+ab-c-ac+bc+abc
  - $Contrast_{AB}$ = (1)-a-b+ab+c-ac-bc+abc
- 各因子之效用 $Effect = Contrast / (n \times 2^{k-1})$
- 各因子之 $Sum\ of\ Square = (Contrast)^2 / (n \times 2^k)$
  - $SS_A = \{-(1)+a-b+ab-c+ac-bc+abc\}^2 / 8n$

Example(10-7) for 2^3 Design

- A: 速度
- B: 切割深度
- C: 切刀角度

Surface-finish data for

Runs		Design Factors A	B	C	n=2 Surface Finish	Totals
1	(1)	−1	−1	−1	9, 7	16
2	*a*	1	−1	−1	10, 12	22
3	*b*	−1	1	−1	9, 11	20
4	*ab*	1	1	−1	12, 15	27
5	*c*	−1	−1	1	11, 10	21
6	*ac*	1	−1	1	10, 13	23
7	*bc*	−1	1	1	10, 8	18
8	*abc*	1	1	1	16, 14	30

計算_Example

- 各因子之效用

速度 $A = \frac{1}{4n}[a + ab + ac + abc - b - c - bc - (1)]$

$= \frac{1}{4(2)}[22 + 27 + 23 + 30 - 20 - 21 - 18 - 16]$

$= \frac{1}{8}[27] = 3.375$

切割深度 $B = 1.625$

切刀角度 $C = 0.875$

$AB = 1.375$

$AC = 0.125$

$BC = -0.625$

$ABC = 1.125$

- 各因子之 *Sum of Square*
  - SS_A = {-(1)+a-b+ab-c+ac-bc+abc}^{2} / 8n = $(27)^2$ / (8×2) = 45.5625

ANOVA 表_Example

Analysis of variance for the surface-finish experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A	45.5625	1	45.5625	18.69*
B	10.5625	1	10.5625	4.33*
C	3.0625	1	3.0625	1.26
AB	7.5625	1	7.5625	3.10
AC	0.0625	1	0.0625	0.03
BC	1.5625	1	1.5625	0.64
ABC	5.5625	1	5.5625	2.08
Error	19.5000	8	2.4375	
Total	92.9375	15		

$F_{0.10,1,8} = 3.46$,

*表顯著

Example for 2^4 Design

The 2^4 design for the plasma etch experiment

蝕刻率(y_i)

Run	A (Gap)	B (Pressure)	C (C_2F_6 Flow)	D (Power)	Etch Rate (Å/min)
1	−1	−1	−1	−1	550
2	1	−1	−1	−1	669
3	−1	1	−1	−1	604
4	1	1	−1	−1	650
5	−1	−1	1	−1	633
6	1	−1	1	−1	642
7	−1	1	1	−1	601
8	1	1	1	−1	635
9	−1	−1	−1	1	1037
10	1	−1	−1	1	749
11	−1	1	−1	1	1052
12	1	1	−1	1	868
13	−1	−1	1	1	1075
14	1	−1	1	1	860
15	−1	1	1	1	1063
16	1	1	1	1	729

2^4因子階層設計_符號表

Contrast constants for the 2^4 design

Run		A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD
1	(1)	−	−	+	−	+	+	−	−	+	+	−
2	a	+	−	−	−	−	+	+	−	−	+	+
3	b	−	+	−	−	+	−	+	−	+	−	+
4	ab	+	+	+	−	−	−	−	−	−	−	−
5	c	−	−	+	+	−	−	+	−	+	+	−
6	ac	+	−	−	+	+	−	−	−	−	+	+
7	bc	−	+	−	+	−	+	−	−	+	−	+
8	abc	+	+	+	+	+	+	+	−	−	−	−
9	d	−	−	+	−	+	+	−	+	−	−	+
10	ad	+	−	−	−	−	+	+	+	+	−	−
11	bd	−	+	−	−	+	−	+	+	−	+	−
12	abd	+	+	+	−	−	−	−	+	+	+	+
13	cd	−	−	+	+	−	−	+	+	−	−	+
14	acd	+	−	−	+	+	−	−	+	+	−	−
15	bcd	−	+	−	+	−	+	−	+	−	+	−
16	$abcd$	+	+	+	+	+	+	+	+	+	+	+

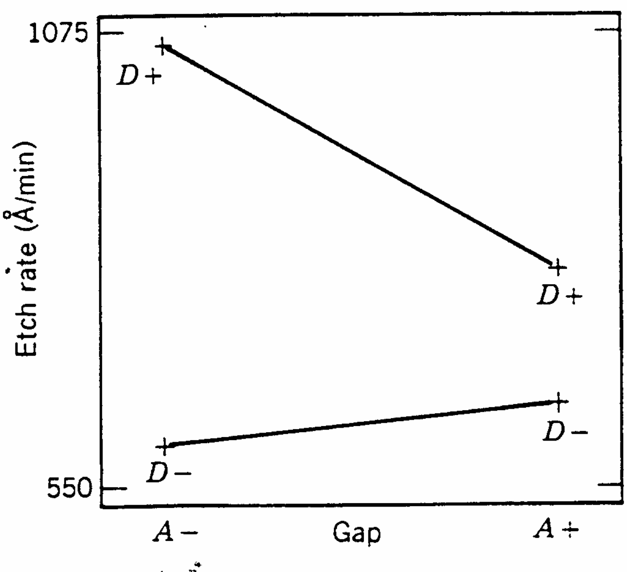
→ 請完成

ANOVA 表_Example

Analysis of variance for the plasma etch experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A	41,310.563	1	41,310.563	20.28 ✓
B	10.563	1	10.563	<1
C	217.563	1	217.563	<1
D	374,850.063	1	374,850.063	183.99 ✓
AB	248.063	1	248.063	<1
AC	2,475.063	1	2,475.063	1.21
AD	94,402.563	1	99,402.563	48.79 ✓
BC	7,700.063	1	7,700.063	3.78
BD	1.563	1	1.563	<1
CD	18.063	1	18.063	<1
Error	10,186.815	5	2,037.363	
Total	531,420.938	15		

AD交互作用與迴歸函數



$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 A + \hat{\beta}_2 D + \hat{\beta}_3 AD$$

$\hat{\beta}_1$ 爲A之效用/2

$\hat{\beta}_2$ 爲D之效用/2

$\hat{\beta}_3$ 爲AD之效用/2

2^k Design with Center Points

- 增加預估曲線作用之能力
- 不破壞設計之平衡性(Balanced Design)
- 只需增加少數幾個實驗

$$SS_{pure\ quadratic} = \frac{n_F n_c (\bar{y}_F - \bar{y}_c)}{n_F + n_c}$$

Example

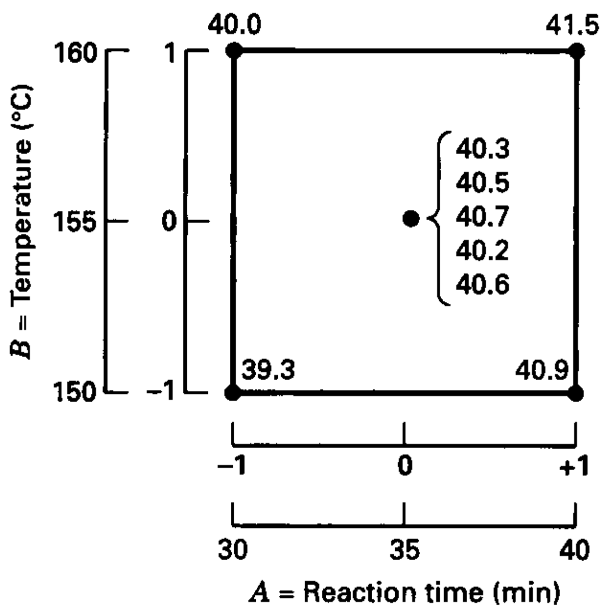


Figure 6-35 The 2^2 design with five center points for Example 6-6.

ANOVA表_Example

Table 6-20 Analysis of Variance for Example 6-6

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
A (Time)	2.4025	1	2.4025	55.87	0.0017
B (Temperature)	0.4225	1	0.4225	9.83	0.0350
AB	0.0025	1	0.0025	0.06	0.8185
Pure quadratic	0.0027	1	0.0027	0.06	0.8185
Error	0.1720	4	0.0430		
Total	3.0022	8			

2^k 因子實驗之區集與交絡

- 2^2 factorial design with Blocking

Table 7-1 Chemical Process Experiment in Three Blocks

	Block 1	Block 2	Block 3
	$(1) = 28$	$(1) = 25$	$(1) = 27$
	$a = 36$	$a = 32$	$a = 32$
	$b = 18$	$b = 19$	$b = 23$
	$ab = 31$	$ab = 30$	$ab = 29$
Block totals:	$B_1 = 113$	$B_2 = 106$	$B_3 = 111$

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The ANOVA

Table 7-2 Analysis of Variance for the Chemical Process Experiment in Three Blocks

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Blocks	6.50	2	3.25		
A (concentration)	208.33	1	208.33	50.32	0.0004
B (catalyst)	75.00	1	75.00	18.12	0.0053
AB	8.33	1	8.33	2.01	0.2060
Error	24.84	6	4.14		
Total	323.00	11			

$$SS_{\text{Blocks}} = \sum_{i=1}^{3} \frac{B_i^2}{4} - \frac{y_{...}^2}{12}$$

$$= \frac{(113)^2 + (106)^2 + (111)^2}{4} - \frac{(330)^2}{12}$$

$$= 6.50$$

Confounding(交絡)

- 受限於資源（時間、金錢、人力等），無法在每一個區集有完整的因子實驗。
- Confounding is a design technique for arranging a complete factorial design in blocks, where the block size is smaller than the number of treatment combinations in one replicate.
- It causes information about certain treatment effects (usually high-order interactions) to be *indistinguishable from*, or *confounded with*, ***blocks***.

Confounding in 2 blocks

- Effect AB confounded with block.

Table 7-3 Table of Plus and Minus Signs for the 2^2 Design

Treatment Combination	Factorial Effect			
	I	A	B	AB
(1)	+	−	−	+
a	+	+	−	−
b	+	−	+	−
ab	+	+	+	+

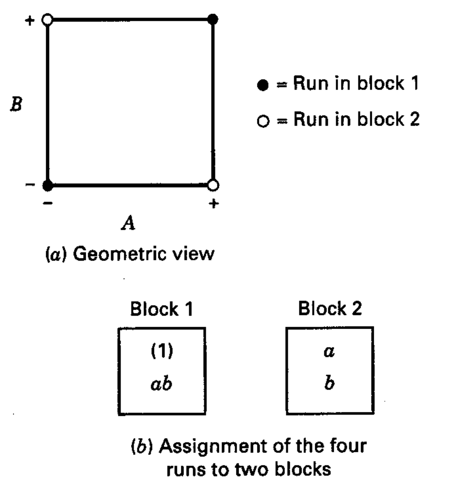


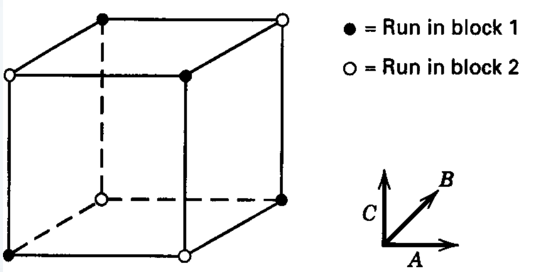
Figure 7-1 A 2^2 design in two blocks.

The 2^3 Design Confounded(I)

Table 7-4 Table of Plus and Minus Signs for the 2^3 Design

Treatment Combination	Factorial Effect							
	I	A	B	AB	C	AC	BC	ABC
(1)	+	−	−	+	−	+	+	−
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
ab	+	+	+	+	−	−	−	−
c	+	−	−	+	+	−	−	+
ac	+	+	−	−	+	+	−	−
bc	+	−	+	−	+	−	+	−
abc	+	+	+	+	+	+	+	+

The 2^3 Design Confounded(II)



(a) Geometric view

Block 1	Block 2
(1)	*a*
ab	*b*
ac	*c*
bc	*abc*

(b) Assignment of the eight runs to two blocks

Figure 7-2 The 2^3 design in two blocks with *ABC* confounded.

- The Degrees of Freedom