Optimal burn-in decision for products with an unimodal failure rate function

卓益如

Abstract

- Some lifetime distribution have an unimodal failure rate function, where a critical time separates product's operating life into an increasing failure rate (IFR) phase and decreasing failure rate (DFR) phase.
- The customer may encounter a risk of high failure both in IFR and DFR phase during the early operating life of products.
- How to eliminate IFR phase economically determine the optimal burn-in time during DFR phase, and incorporate the MRL into consideration are essential to the burn-in decision.

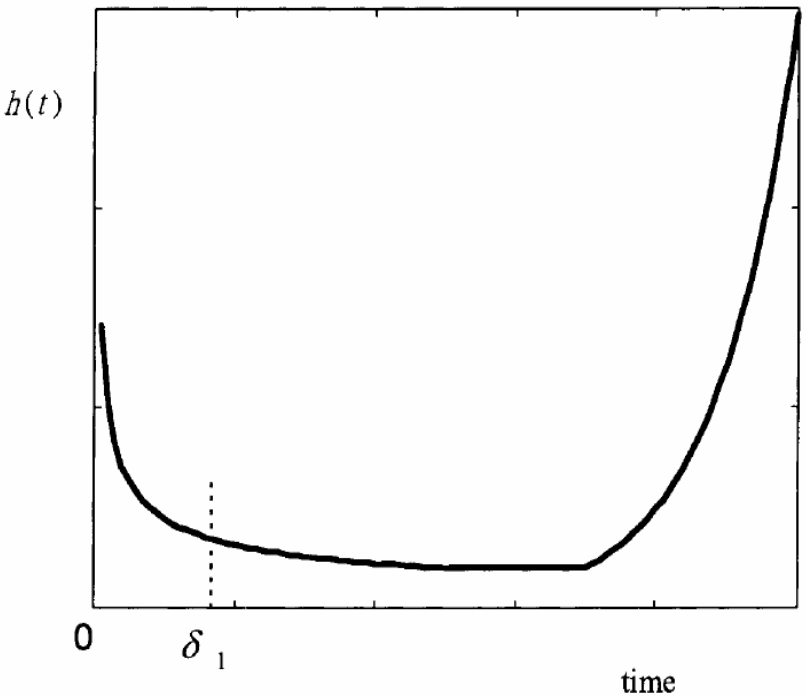
Key word

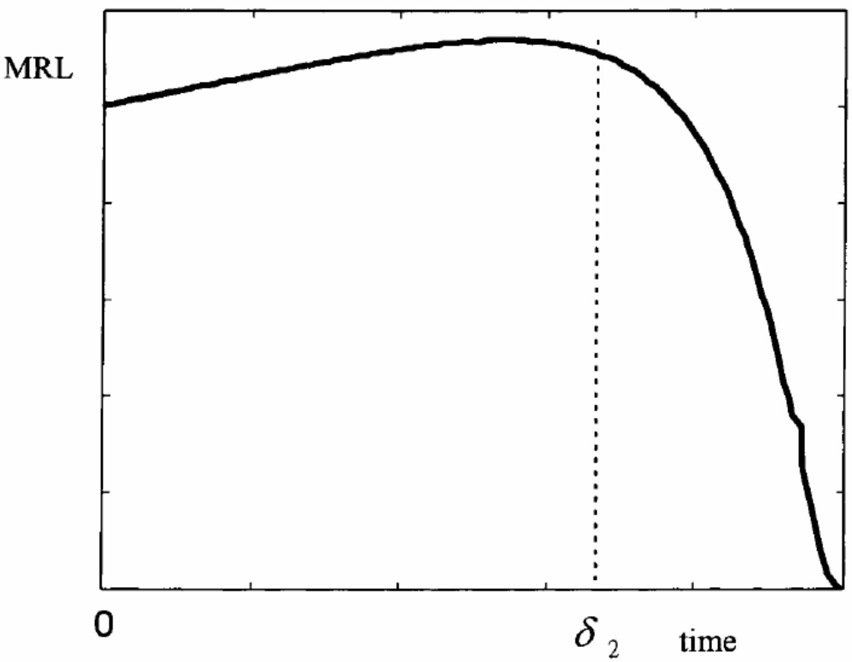
- Reliability
- Burn-in
- Critical time
- Mean residual life
- Constrained optimization

The text

- Cost configurations affecting the determination of burn-in time include the burn-in cost, failure cost during burn-in, and warranty cost.
- The optimal burn-in time is determined to minimize the expected total cost.
- Another manner is to maximize the MRL for bathtub-shaped failure rate function.

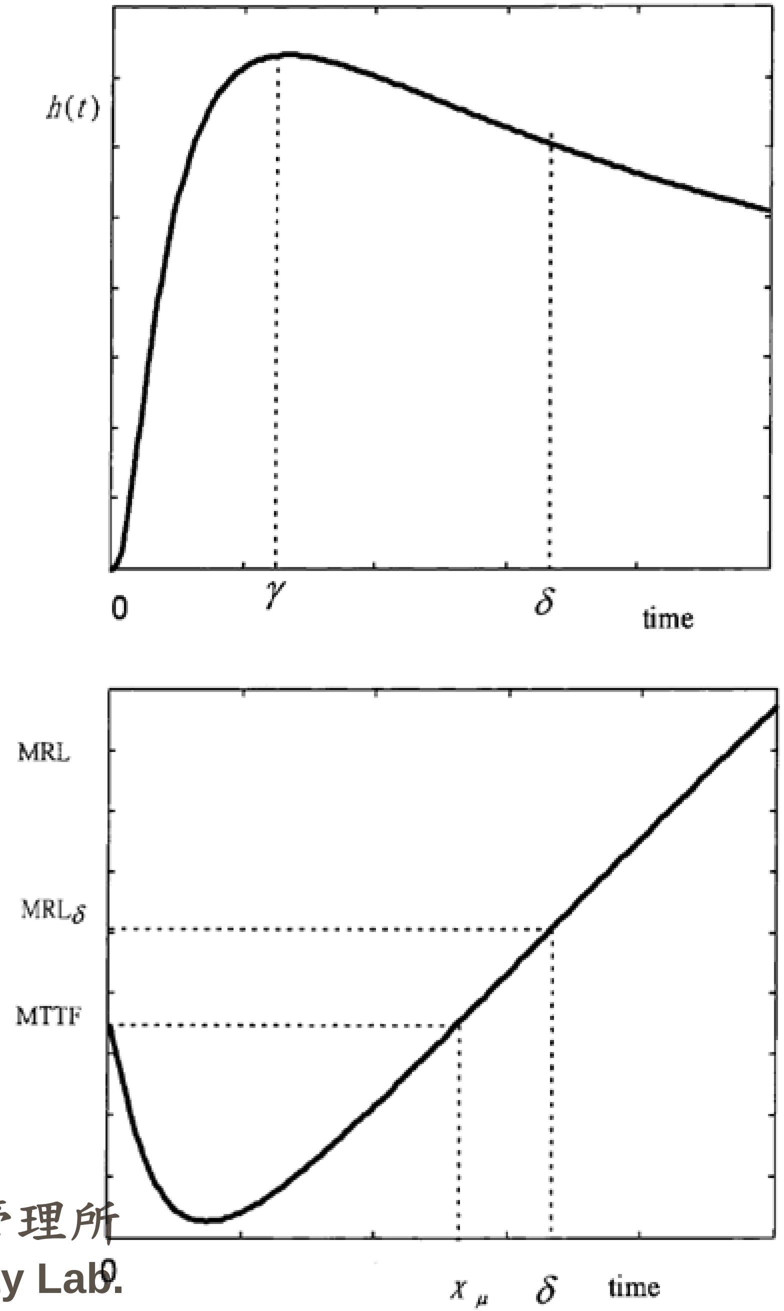
Burn-in decision for products with a bathtub-shaped failure rate function.





- In the situation of an unimodal FR function, Chang and Su suggested to burn-in at least past the critical time to avoid the highest hazard rate.
- As the MRL is upside-down unimodal, maximizing the MRL can’t be the goal of optimal burn-in decision since the MRL is unbounded in extreme point.

Burn-in decision for products with an unimodal failure rate function.



The notations：

T lifetime

$R(t)$ reliability function

$h(t)$ failure rate function

$\text{MRL}(t)$ mean residual life function

γ critical time of $h(t)$

τ change - point of $h(t)$

δ burn - in time

$f(t;\underline{\theta})$ probability density function of lifetime T with parameter vector θ

p fraction of mean time to failure

T_w length of the warranty period

$v(\delta,\gamma)$ per - item manufacturing cost

$\omega(Tw,\delta,\gamma)$ per - item warranty cost

c_1 manufacturing cost per item without burn - in

c_2 cost of burn - in per item unit time

c_3 repair cost per failure

c_4 extra cost that arises when a failure occurs during the warranty period

X_u the time when the mean residual life after burn - in equals to its mean time to failure

$\phi(z)(\Phi(z))$ probability density function of standard normal distribution

μ log(median life) of lognormal distribution

σ standard deviation of lognormal distribution

Burn-in model

$$h(t)$$

$$\begin{cases} \text{strictly increase} & \text{for } 0 \le t < \gamma \\ \text{reaches the highest value, say } h^* \text{ at } t = r \\ \text{strictly decreases} & \text{for } \gamma < t < \tau \\ \text{is approximately a constant} & \text{for } \tau \le t \le \infty \end{cases}$$

$$h(t;\theta) - q(t;\theta) = 0 \quad \text{, where} \quad q(t;\theta) = \frac{-f'(t;\theta)}{f(t;\theta)} \qquad (1)$$

$$r < \delta \le p \cdot MTTF \qquad (2)$$

$$\text{MRL}(\delta) \ge \text{MTTF} \qquad (3)$$

$$\text{here } \text{MRL}(t) = E[T - t \mid T > t]$$

$$= \frac{1}{R(t)} \int_t^{\infty} y \cdot f(y)\,dy - t$$

$$C(Tw, \delta, \gamma) = v(\delta, \gamma) + \omega(Tw, \delta, \gamma) \qquad (4)$$

$$v(\delta, \gamma) = c_0 + c_1 + c_2\delta + c_3 \int_0^{\delta} h(t)\,dt \qquad (5)$$

$$\omega(Tw, \delta, \gamma) = (c_3 + c_4) \int_0^{Tw} h(t + \delta)\,dt \qquad (6)$$

NLP model：

Minimize

$$C(T_W,\delta,\gamma)=c_0+c_1+c_2\delta+c_3\int_0^{\delta}h(t)\mathrm{d}t+(c_3+c_4)\int_0^{T_W}h(t+\delta)\mathrm{d}t$$

subject to

$$\frac{1}{R(\delta)}\int_{\delta}^{\infty}y\cdot f(y)\mathrm{d}y-\delta\geqslant \mathrm{MTTF},$$

$$\gamma<\delta\leqslant p\cdot\mathrm{MTTF},\quad \delta>0,\quad \gamma>0$$

$$C(Tw,\delta,\gamma)=c_0+c_1+c_2\delta+c_3\int_0^{\delta}h(t)dt+(c_3+c_4)\left[\int_0^{\tau-\delta}h(t+\delta)dt+\int_{\tau-\delta}^{Tw}h(t+\delta)dt\right]$$

$$=c_0+c_1+c_2\delta+(c_3+c_4)\left[\int_0^{\tau-\delta}h(t+\delta)dt+\int_{\tau-\delta}^{Tw}h(t+\delta)dt+\int_0^{\delta}h(t)dt\right]-c_4\int_0^{\delta}h(t)dt$$

$$\frac{dC(Tw,\delta,\gamma)}{d\delta}=c_2-c_4h(\delta)+(c_3+c_4)h(\tau)$$

$$\frac{d^2C(Tw,\delta,\gamma)}{d\delta^2}=-c_4h'(\delta)>0$$

- Investigate the individual effect of parameter on the critical time γ and X_u=MRL^{-1}(MTTF)
- Plot the trajectories of $\gamma(\theta)$ 、$X_u(\theta)$ and MTTF(θ)
- Compare γ and X_u with MTTF respectively.
- Feasible range of the parameter：

$$p \cdot MTTF(\theta) > \delta > \gamma(\theta) \text{ if } r(\theta) > Xu(\theta)$$

or

$$p \cdot MTTF(\theta) > \delta > Xu(\theta) \text{ if } Xu(\theta) \geq r(\theta)$$

- Search the optimal burn-in time using the proposed cost model.

Conclusion

- This paper extends Chang and Su's model to obtain a more realistic burn-in decision for the case of unimodal failure rate function.
- By investigating the relationship among the critical time, X_u and MTTF，the feasible range of parameters for conducting an effective burn-in can be addressed.