Design of double- and triple-sampling X-bar control chart using genetic algorithms

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Introduction

- Brief background information on traditional Shewhart X-bar control chart and DS X-bar control charts is provided.
- It’s devoted to the methods for constructing the DS and TS X-bar control charts.
- Computational results in comparing the efficiency of TS with DS charts are provided.

Shewhart X-bar control chart

- Suppose that the mean of *x* is μ and the standard deviation (SD) is σ
- The following three parameters that characterize the Shewhart X-bar control chart are:

  $UCL = \mu + k\sigma \quad LCL = \mu - k\sigma,$

  where UCL is the upper control limit, CL is the centre line or the process average, LCL is the lower control limit and *k* is the distance of the control limits from the centre line and is expressed as a multiple of σ.

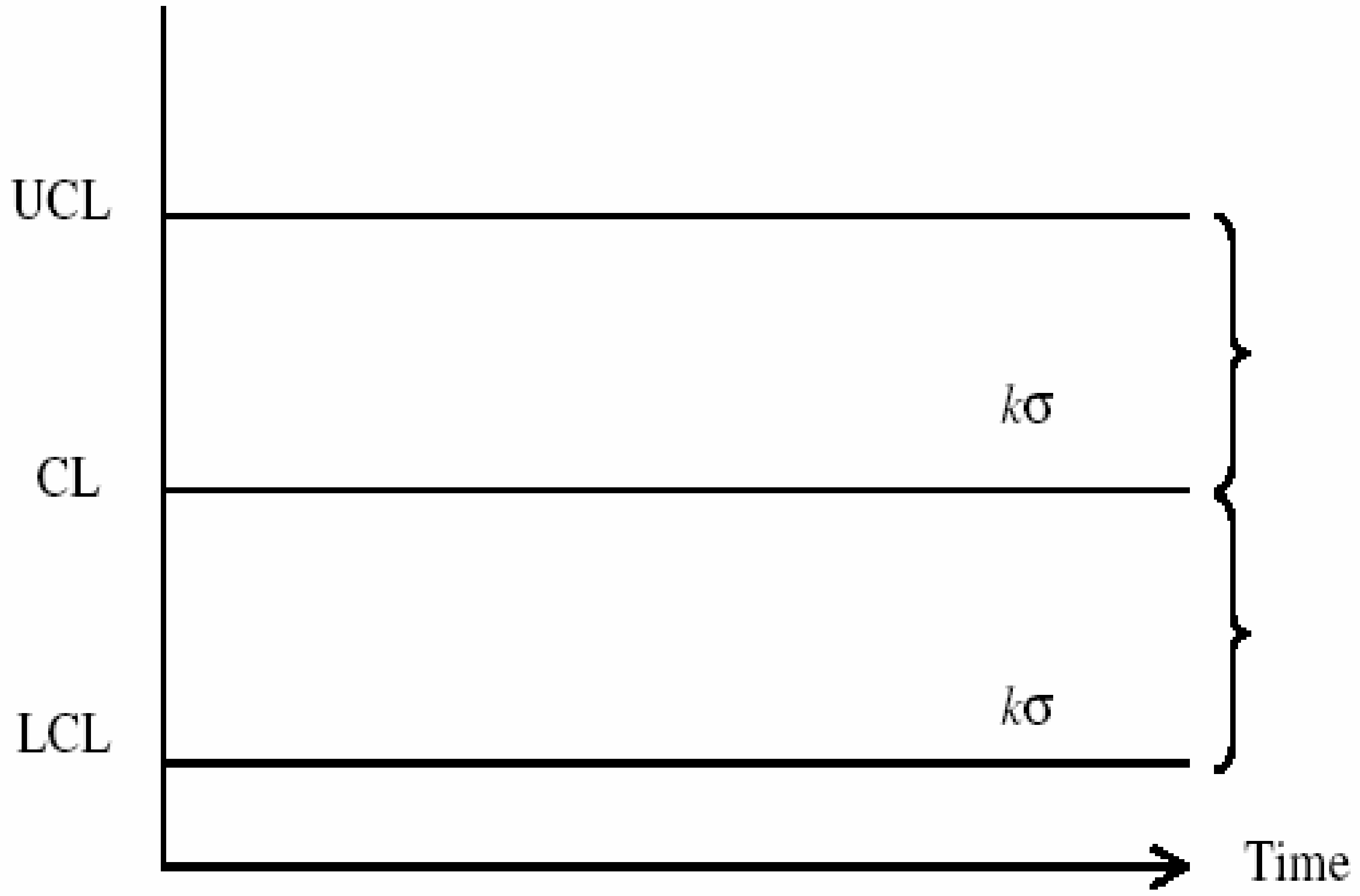

Figure 1. Shewhart X-bar control chart.

DS X-bar control chart

The design of a DS X-bar chart involves determining five parameters.

- n1 =Sample size of the first sample
- n2 = Sample size of the second sample
- *L*1 and -*L*1= limits on the first sample within which the process is said to be in control
- *L*2 and -*L*2 = limits on the second sample within which the process is said to be in control
- *L* and -*L* = limits on the first sample beyond which the process is said to be out of control

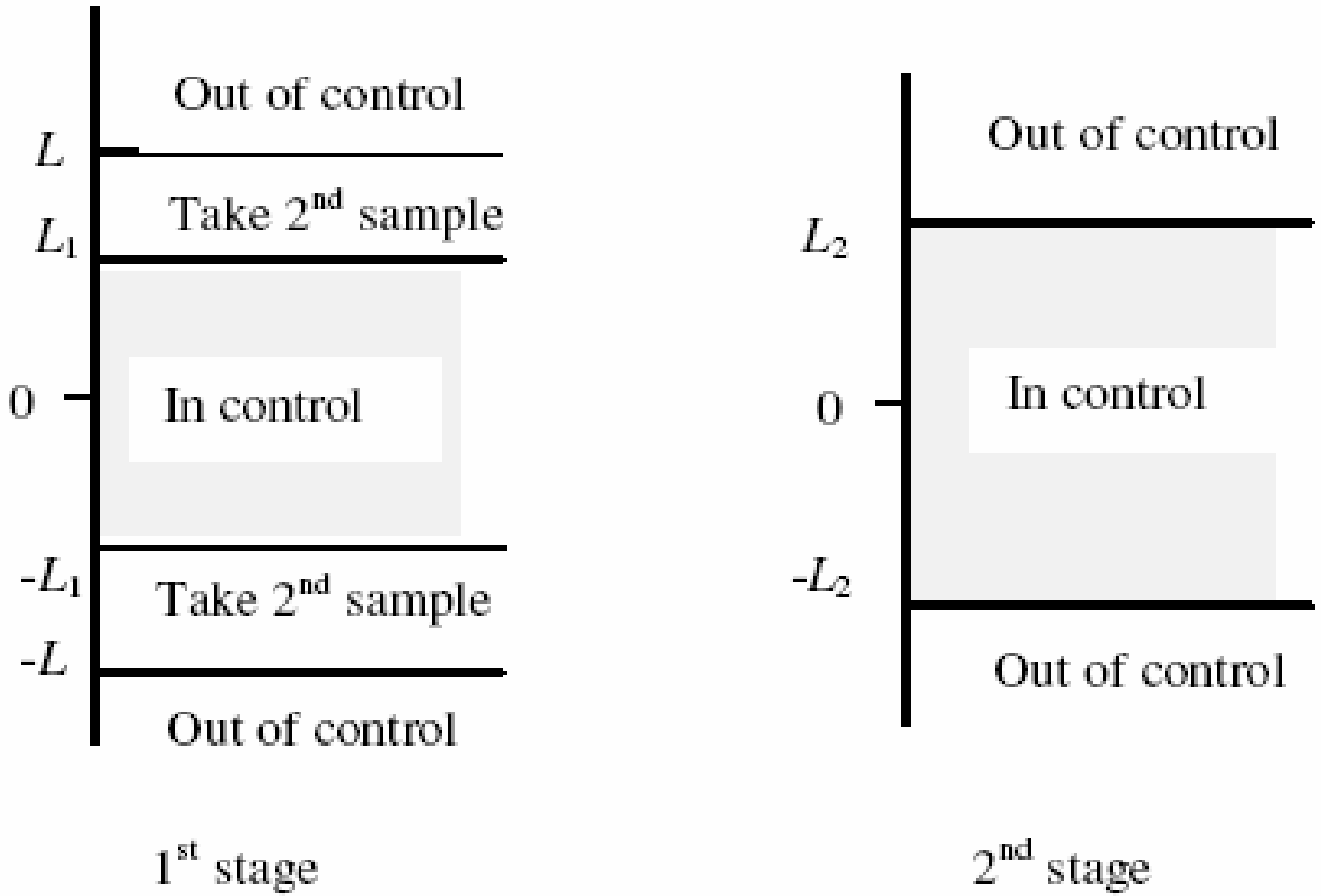

Figure 2. Graphic view of the DS X-bar chart.

2.2.1. *Daudin's DS X-bar control procedure*

(1) Take an initial sample of size n_1. Calculate the mean of the sample $\overline{X_1}$.

(2) If $\dfrac{\overline{X_1}-\mu_0}{\sigma/\sqrt{n_1}}$ lies in the range $[-L_1, L_1]$, the process is in control.

(3) If $\dfrac{\overline{X_1}-\mu_0}{\sigma/\sqrt{n_1}}$ lies in the range $(-\infty, -L]$ or $[L, +\infty)$, the process is out of control.

(4) If $\dfrac{\overline{X_1}-\mu_0}{\sigma/\sqrt{n_1}}$ lies in the range $(-L, -L_1)$ or (L_1, L), take a second sample of size n_2 and calculate the sample mean $\overline{X_2}$.

(5) Calculate the total sample mean $\overline{Y} = \dfrac{n_1\overline{X_1}+n_2\overline{X_2}}{n_1+n_2}$ at the second stage.

(6) If $-L < \dfrac{\overline{X_1}-\mu_0}{\sigma/\sqrt{n_1}} < -L_1$ or $L_1 < \dfrac{\overline{X_1}-\mu_0}{\sigma/\sqrt{n_1}} < L$ and

if $\dfrac{\overline{Y}-\mu_0}{\sigma/\sqrt{n_1+n_2}} < -L_2$ or $\dfrac{\overline{Y}-\mu_0}{\sigma/\sqrt{n_1+n_2}} > L_2$, then the process is out of control,

else the process is in control.

Design of DS X-bar control chart using genetic algorithm

- To solve the problem of an optimal design of a DS X-bar control chart using a systematic and robust method, the optimization problem is reformulated as follows:

$$\min_{n_1, n_2, L, L_1, L_2} n_1 + n_2 P_r[\overline{X_1} \in I_2 | \mu = \mu_0], \tag{1}$$

subject to

$P_r[\text{out of control} | \mu = \mu_0] \leq \alpha$, i.e.

$$1 - \{\Phi[L_1] - \Phi[-L_1]\} - \int_{z \in I_2^*} \left\{ \Phi\left[c\, L_2 - z\sqrt{\frac{n_1}{n_2}}\right] - \Phi\left[-c\, L_2 - z\sqrt{\frac{n}{n_2}}\right] \right\} \varphi(z)\, \mathrm{d}z \leq \alpha. \tag{2}$$

$P_r[\text{in control} | \mu = \mu_1] \leq \beta$ (for a given intended shift δ), i.e.

$$\{\Phi[L_1 + \delta\sqrt{n_1}] - \Phi[-L_1 + \delta\sqrt{n_1}]\} + \int_{z \in I_2^*} \left\{ \Phi\left[cL_2 + rc\delta - z\sqrt{\frac{n_1}{n_2}}\right] - \Phi\left[-cL_2 + rc\delta - z\sqrt{\frac{n_1}{n_2}}\right] \right\} \varphi(z)\, \mathrm{d}z \leq \beta \tag{3}$$

$$n_i \geq 1, \text{integer}, \text{for } i = 1, 2 \tag{4}$$

$$L \geq L_l \tag{5}$$

$$L_{1l} \leq L_1 \leq L_{1u} \tag{6}$$

$$L_{2l} \leq L_2 \leq L_{2u} \tag{7}$$

Implementation of GA

- The optimization model (1-7) is set up in an Excel spreadsheet and solved by the GA in Evolver.
- The operation of the GA involves following steps:

(1) Create a random initial solution;

(2) Evaluate fitness, i.e. the objective function that minimizes the average sample size when the process is in control;

(3) Reproduction and mutation; and

(4) Generate new solutions.

Developed of TS X-bar control chart

The design of a TS X-bar control chart involves

- determining the following parameters
- n1 = Sample size of the first sample
- n2 = Sample size of the second sample
- n3 = Sample size of the third sample
- L, $L1$ = limits at the first stage
- $L2$, $L3$ = limits at the second stage
- $L4$ = limits at the third stage

The following procedure is proposed.

(1) Take an initial sample of size n1. Calculate the sample mean $\overline{X}_1$.

(2) If $\frac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}}$ lies in the range $[-L_1, L_1]$, the process is in control.

(3) If $\frac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}}$ lies in the range $(-\infty, -L]$ or $[L, +\infty)$, the process is out of control.

(4) If $\frac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}}$ lies in the range $(-L, -L_1)$ or (L_1, L), take a second sample of size n_2 and calculate the sample mean $\overline{X}_2$

(5) Calculate the total sample mean $\overline{Y} = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2}$ at the second stage.

(6) If $-L < \frac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}} < -L_1$ or $L_1 < \frac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}} < L$, and if $\frac{\overline{Y} - \mu_0}{\sigma/\sqrt{n_1 + n_2}} < -L_3$ or $\frac{\overline{Y} - \mu_0}{\sigma/\sqrt{n_1 + n_2}} > L_3$, then the process is out of control.

(7) If $-L < \dfrac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}} < -L_1$ or $L_1 < \dfrac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}} < L$, and

$-L_2 < \dfrac{\overline{Y} - \mu_0}{\sigma/\sqrt{n_1 + n_2}} < L_2$, then the process is in control.

(8) If $-L < \dfrac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}} < -L_1$ or $L_1 < \dfrac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}} < L$, and if

$-L_3 < \dfrac{\overline{Y} - \mu_0}{\sigma/\sqrt{n_1 + n_2}} < -L_2$ or $L_2 < \dfrac{\overline{Y} - \mu_0}{\sigma/\sqrt{n_1 + n_2}} < L_3$, then take the third sample of size n_3 and calculate the sample mean $\overline{X}_3$.

(9) Calculate the total sample mean $\overline{W} = \dfrac{n_1\overline{X}_1 + n_2\overline{X}_2 + n_3\overline{X}_3}{n_1 + n_2 + n_3}$ at the third stage.

(10) If $-L < \dfrac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}} < -L_1$ or $L_1 < \dfrac{\overline{X}_1 - \mu_0}{\sigma/\sqrt{n_1}} < L$, and if

$-L_3 < \dfrac{\overline{Y} - \mu_0}{\sigma/\sqrt{n_1 + n_2}} < -L_2$ or $L_2 < \dfrac{\overline{Y} - \mu_0}{\sigma/\sqrt{n_1 + n_2}} < L_3$, and if

$\dfrac{\overline{W} - \mu_0}{\sigma/\sqrt{n_1 + n_2 + n_3}} < -L_4$ or $\dfrac{\overline{W} - \mu_0}{\sigma/\sqrt{n_1 + n_2 + n_3}} > L_4$, then the process is out of control; else the process is in control.

A TS X-bar chart is given in figure 3.

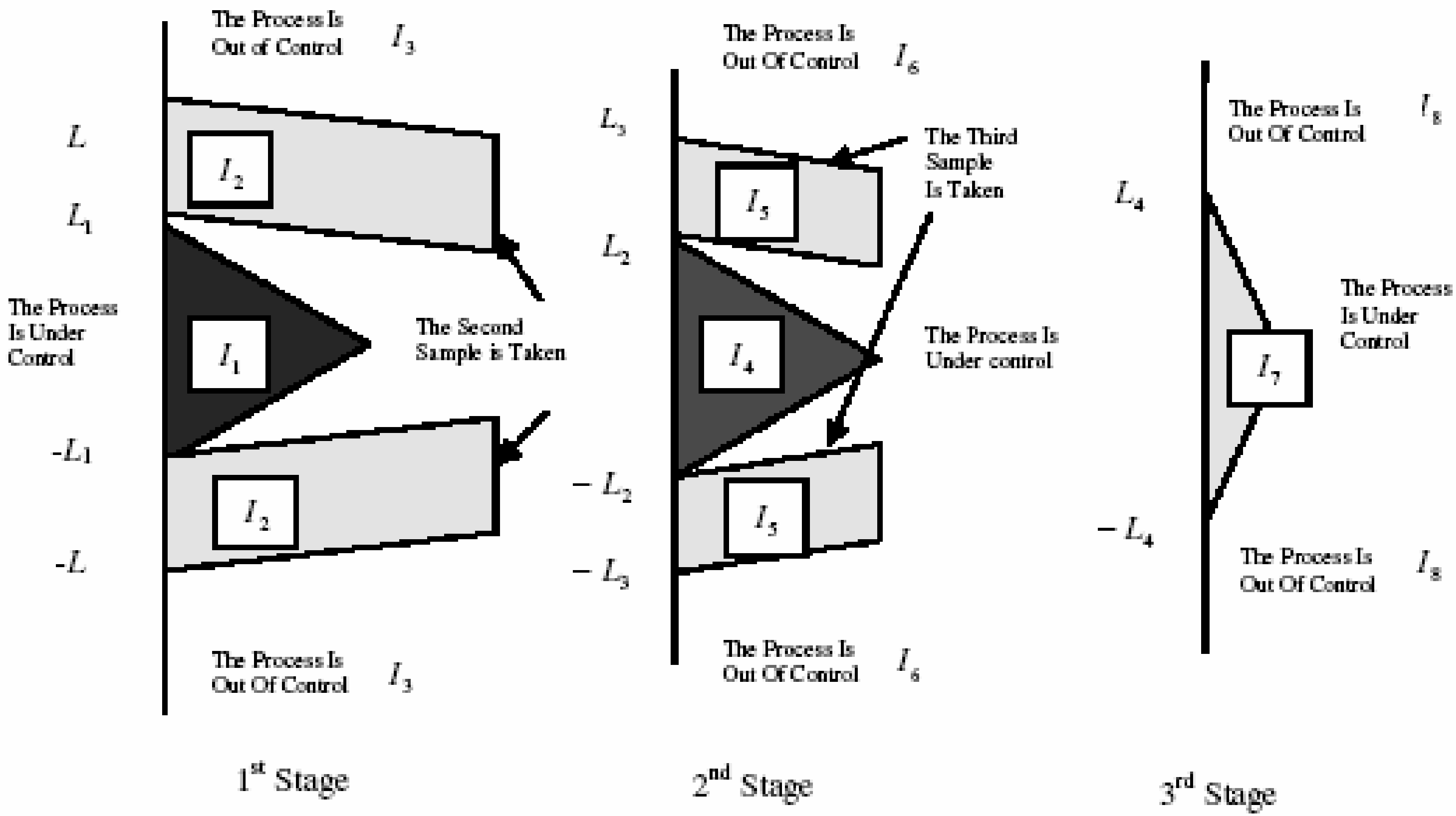

Figure 3. Graphic view of the TS X-bar control chart.

Comparison of TS X-bar chart with DS X-bar chart

- The decreased percentage of the expected average sample size was calculated using:

$$\text{Decrease} = \left(\frac{E(N)_{DS} - E(N)_{TS}}{E(N)_{DS}}\right) \times 100\% .$$

Chart type	ARL_0	$ARL_1(\delta)$	n_1	n_2	n_3	L_1	L	L_2	L_3	L_4	$E(N)$	Decease %
DS	370.4	1.186(2.83)	1	2	–	1.81	5.0	2.77	–	–	1.14	1.8
TS			1	1	1	1.62	3.07	1.8	3.35	2.86	1.12	
DS	500.0	1.222(2.83)	1	2	–	1.90	5.0	2.85	–	–	1.12	3.6
TS			1	1	1	1.79	3.0	1.8	3.01	2.93	1.08	
DS	370.4	1.186(2)	2	3	–	1.74	5.0	2.85	–	–	2.25	7.6
TS			2	1	1	1.76	3.00	1.8	3.69	2.66	2.08	
DS	500.0	1.222(2)	2	3	–	1.82	5.0	2.94	–	–	2.21	2.7
TS			2	2	1	1.80	3.00	1.8	3.39	2.85	2.15	
DS	370.4	1.186(1.79)	2	4	–	1.37	5.0	2.90	–	–	2.68	13.8
TS			2	2	1	1.47	3.00	1.8	3.3	2.87	2.31	
DS	500.0	1.222(1.79)	2	5	–	1.45	5.0	2.98	–	–	2.59	9.3
TS			2	2	2	1.49	3.00	1.47	4.51	2.81	2.35	
DS	370.4	1.186(1.63)	2	6	–	1.22	5.0	2.87	–	–	3.33	20.7
TS			2	2	3	1.23	3.32	1.55	3.90	2.97	2.64	
DS	500.0	1.222(1.63)	2	6	–	1.31	5.0	2.96	–	–	3.14	16.6
TS			2	2	3	1.34	3.67	1.56	3.14	2.88	2.62	
DS	370.4	1.186(1.51)	3	6	–	1.61	5.0	2.86	–	–	3.76	10.6
TS			3	3	2	1.57	3.00	1.8	3.61	2.81	3.36	
DS	500.0	1.222(1.51)	3	6	–	1.61	5.0	2.96	–	–	3.64	9.3
TS			3	3	2	1.66	3.0	1.8	3.86	2.87	3.30	

DS	370.0	1.186(1.41)	3	7	–	1.32	5.0	2.88	–	–	4.30	3.0
TS			3	3	4	1.41	3.0	1.61	4.07	2.86	4.17	
DS	500.0	1.222(1.41)	3	7	–	1.41	5.0	2.97	–	–	4.11	11.7
TS			3	3	5	1.48	3.17	1.8	3.44	2.89	3.63	
DS	370.4	1.186(1.33)	4	7	–	1.54	5.0	2.88	–	–	4.87	8.2
TS			4	4	3	1.63	3.00	1.66	3149	2.84	4.47	
DS	500.0	1.222(1.33)	3	9	–	1.31	5.0	2.95	–	–	4.72	18.9
TS			3	3	4	1.32	3.72	1.68	3.56	2.82	3.83	
DS	370.4	1.186(1.26)	4	9	–	1.44	5.0	2.86	–	–	5.53	10.8
TS			4	4	6	1.49	3.13	1.78	3.09	2.91	4.77	
DS	500.0	1.222(1.26)	4	9	–	1.53	5.0	2.95	–	–	5.14	10.1
TS			4	5	4	1.59	3.0	1.8	3.39	2.97	4.62	
DS	370.4	1.186(1.15)	5	10	–	1.47	5.0	2.87	–	–	6.42	12.8
TS			5	8	3	1.55	3.0	1.8	3.57	2.89	5.60	
DS	500.0	1.222(1.15)	5	10	–	1.55	5.0	2.96	–	–	6.21	5.0
TS			5	5	6	1.43	3.36	18	3.81	2.98	5.90	
DS	370.0	1.186(0.89)	8	17	–	1.41	5.0	2.88	–	–	10.69	11.3
TS			8	10	5	1.49	3	1.67	3.18	2.72	9.48	
DS	500.0	1.222(0.89)	8	18	–	1.53	5.0	2.95	–	–	10.28	15.56
TS			8	5	7	1.54	3.085	1.71	3.74	2.76	8.68	

Table 1. Results of a comparison between the TS and DS charts.

Conclusions

- The results showing that TS charts were more efficient in terms of minimizing the average sample size. There is a decrease in the expected average sample size for the TS control scheme compared with the DS scheme.
- The TS scheme requires more time for decision-making than the DS scheme since the number of the sampling stage is larger. For cases where the statistical efficiency is more important than the time required for decision-making.