On the optimal selection of process alternatives in a Six Sigma implementation

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出處:International journal of production economics

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- 六標準差是近年來用來降低不良率的方法，很多公司使用六標準差提升競爭力。

Table 1
Rating of process improvement techniques (Dusharme, 2006)

Process improvement tool	Impact (%)
Six Sigma	53.60
Process mapping	35.30
Root cause analysis	33.50
Cause and effect analysis	31.30
ISO 9001	21.00
Statistical process control	20.10
Total quality management	10.30
Malcolm Baldridge criteria	9.80
Knowledge management	5.80

- 六標準差可以說是全面品質管理(TQM)的延伸，但它與TQM最大的不同是它著重企業整體改善而TQM大多為了提升投資報酬率。
- 六標準差設計以公司中心設計工序為核心，旨在促進最高的增長率。強有力的第一步是傾聽客戶的意見，將其轉換成設計中具體量化規格，然後用最佳的工具生產最好的產品，提供最好的服務，達到客戶的期望。

Table 3
Benefits of Six Sigma in manufacturing sector

Company/project	Metric/measures	Benefits/savings
Motorola (1992)	In-process defect levels	150 times reduction
Raytheon/aircraft integration system	Depot maintenance inspection time (measured in days)	Reduced 88%
GE/Railcar leasing business	Turnaround time at repair shops	62% reduction
Allied Signal (Honeywell)/ laminates plant in South Carolina	Capacity/Cycle time/Inventory/On-time delivery	Up 50%/Down 50%/Down 50%/Increased to near 100%
Allied Signal (Honeywell)/Bendix IQ brake pads	Concept to shipment cycle time	Reduced from 18 months to 8 months)
Hughes aircraft's missile systems group/wave soldering operations	Quality/productivity	Improved 1000%/improved 500%
Continental Teves/Brake and axle assemblies	Failure rate	More than 50% reduction in failure rate
Borg Warner Turbo Systems	Financial	$ 1.5 million annually since 2002
General electric	Financial	$2 Billion in 1999
Motorola (1999)	Financial	$15 Billion over 11 years
Dow chemaical/rail delivery project	Financial	Savings of $2.45 million in capital expenditures
DuPont/Yerkes plant in New York (2000)	Financial	Savings of more than $25 million
Telefonica de espana (2001)	Financial	Savings and increase in revenue 30 million Euro in first 10 months
Texas instruments	Financial	$600 million
Johnson and Johnson	Financial	$ 500 million
Honeywell	Financial	$ 1.2 billion
Ford motor compant/exterior Surface defects	Financial	$500,000

(*Source*: Kwak and Anbari, 2006; Weiner, 2004; De Feo and Bar-El, 2002; Antony and Banuelas, 2002; Buss and Ivey, 2001; McClusky,

Sigma level

- 考慮改善流程從3 sigma level提升至4 sigma level，那麼產品不良率就會從百萬分之66,811降至6210；從5 sigma level提升至6 sigma level，那麼產品不良率就會從百萬分之233降至3.4。
- 意即，有較低的 sigma level就會產生較多的瑕疵率；有較高的 sigma level就會有較少的瑕疵率。

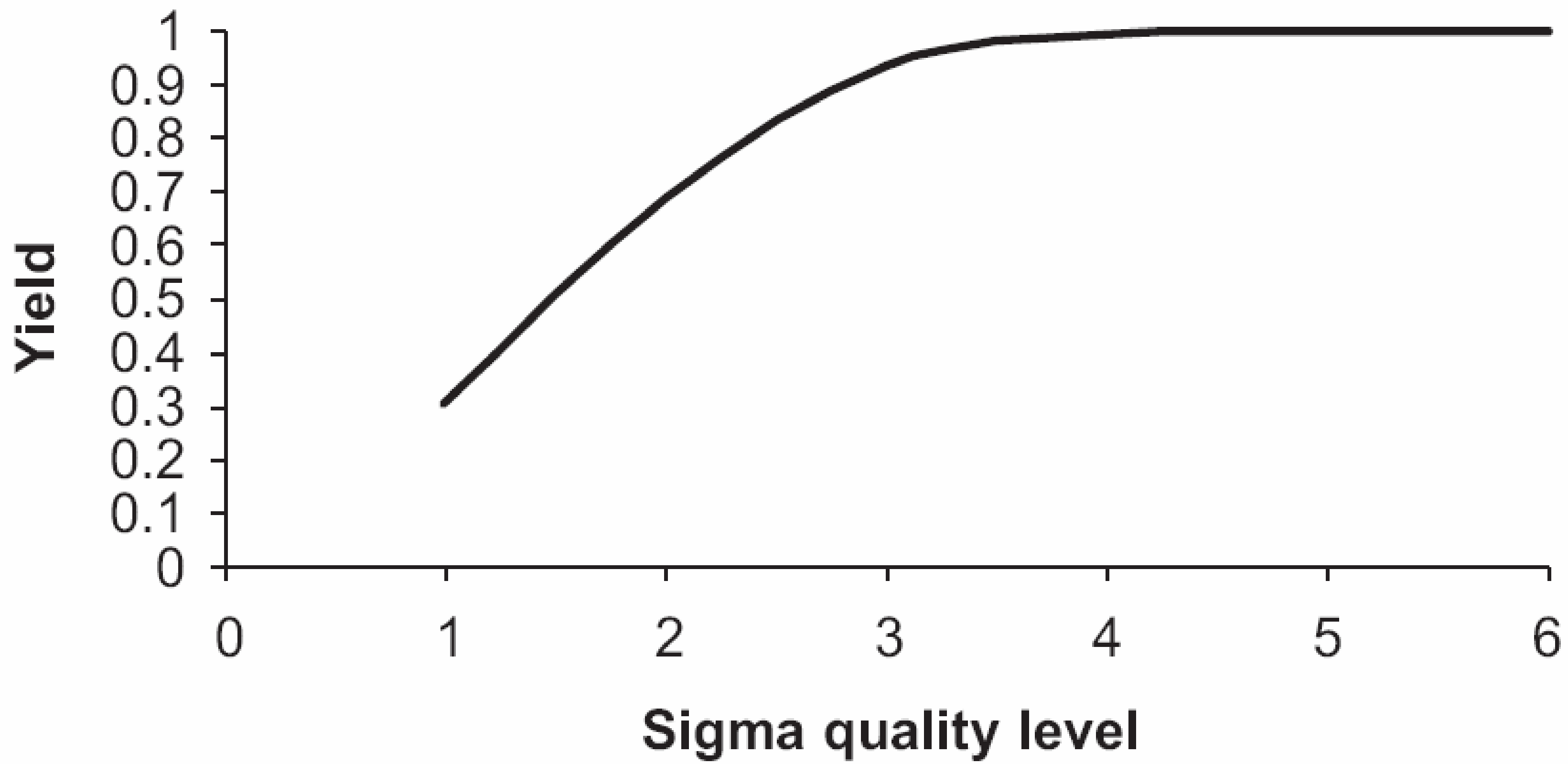

Fig. 1. Decreasing return to scale in Yield as Sigma quality increases.

- 任何sigma level改善均會考慮降低其「不良品質成本」(Cost of Poor Quality) 。不良品質成本包括重工成本、原料使用過多成本、保證成本以及不必要資源使用成本。
- 本論文提及的模式，假設品質良率為一線性函數，而且隨「廢料」(scrap)或「重工」(rework)而改變。
- 對任何目標模式第一步都是選擇最佳執行方案使得sigma level最大化。

符號

Nomenclature

i	ith stage of process, $i = 1, \ldots, n$
Y	Process yield
j	jth defect type for each stage, $j = 1,2$. (1 = scrap, 2 = rework)
k	kth Six Sigma implementation alternative, $k = 1, \ldots, q_i$
f_{ij}	Rate of defect type j at state i
r_{ijk}	Rate of defect type j for alternative k in state i
δ_{ik}	Decision variable related to the kth process alternative for the ith stage
I_i	Number of units that are processed at the ith stage
U	Unit selling price
c_i^m	Marginal production cost of an item at the ith stage
Q_{ik}	Implementation cost for the kth process alternative for the ith stage
S	Raw material available at the first stage
D	Process demand
B	Budget available for Six Sigma alternatives
K_i	Production capacity for the ith production stage
R_i	Amount of rework resulting from the ith stage
c_i^R	Rework cost at the ith stage
c_i^X	Scrap cost at the ith stage
Z^+	Set of positive integers

Model 1：

- 主要探討sigma level和良率(yield)的數學關係。

$$\text{Max } Y = \prod_{i=1}^{n}\left(\sum_{k=1}^{q_i}\delta_{ik}(1-f_{i1}+r_{i1k})\right)$$

Subject to:

I. $$\sum_{k=1}^{q_i}\delta_{ik} = 1\forall i$$

II. $$\sum_{i=1}^{n}\sum_{k=1}^{q_i}\delta_{ik}Q_{ik} \leqslant B$$

III. $I_1 \leqslant S$

IV. $I_i \leqslant K_i \forall i$

$$\text{V.}\, I_{n+1} = D \qquad \text{(where } I_{n+1}=O_n\text{)}$$

$$\text{VI.}\, I_{i+1} = I_i \left(\sum_{k=1}^{q_i} \delta_{ik} \left(1 - f_{i1} + r_{i1k}\right) \right) \forall i$$

$$\text{VII.}\, \delta_{ik} = Bin(0,1) \text{ and } I_i \in Z^+$$

Model 2:

- 以最大化利潤為思考方向，找出最佳執行方案

$$\text{Max Profit} = \overbrace{uI_{n+1}}^{a} - \overbrace{\left[\sum_{i=1}^{n} \left(\sum_{k=1}^{q_i} \delta_{ik} Q_{ik} \right) + \Delta_i + \Omega_i + \Psi_i \right]}^{b}$$

Subject to:

$$\text{I. } \sum_{k=1}^{q_i} \delta_{ik} = 1 \forall i$$

$$\text{II. } \sum_{i=1}^{n} \sum_{k=1}^{q_i} \delta_{ik} Q_{ik} \leqslant B$$

$$\text{III. } I_1 \leqslant S$$

$$\text{IV. } I_i \leqslant K_i \forall_i$$

$$\text{V. } I_{n+1} = D \text{ where, } (I_{n+1} = O_n)$$

$$\text{VI. } I_{i+1} = I_i \left(\sum_{k=1}^{q_i} \delta_{ik} (1 - f_{i1} + r_{i1k}) \right) \forall i$$

Illustrative example

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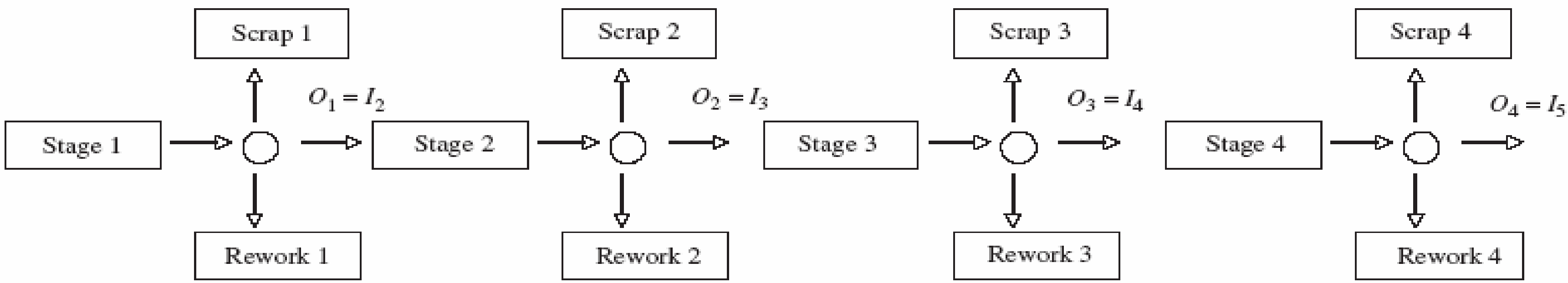

Fig. 3. Four-Stage Process.

※Six sigma 團隊首先應該分析不同選擇方案的好處，再去確定執行

Table 4
Current process rates, costs and capacities

Stage	f_{i1}	$f_{i,2}$	c_i^R	c_i^m	K_i
0[a]	—	—	—	1.0	—
1	0.30	0.05	1.00	2.0	100,000
2	0.08	0.10	1.50	3.0	90,000
3	0.20	0.05	2.00	4.0	80,000
4	0.10	0.10	2.50	5.0	70,000

[a]Stage 0 accounts for the raw material cost needed to start the first process.

Table 5

Reduction in scrap and rework rates and associated cost for each stage improvement alternative

Stage	Alternative[a]											
	0			1			2			3		
	r_{i10}	r_{i20}	Q_{i0}	r_{i11}	r_{i21}	Q_{i1}	r_{i12}	r_{i22}	Q_{i2}	r_{i13}	r_{i23}	Q_{i3}
1	0	0	0	0.030	0.030	40	0.050	0.040	70	0.007	0.007	25
2	0	0	0	0.003	0.003	30	0.060	0.030	85	0.050	0.060	110
3	0	0	0	0.008	0.008	25	0.007	0.007	30	0.007	0.002	15
4	0	0	0	0.060	0.060	110	0.009	0.009	40	0.007	0.007	35

[a] $Q_{ik}10^4$.

Table 6
Optimal values for δ_{ik} under *Model 1* solution

Stage	Alternative[a]			
	0	1	2	3
1	0	1	0	0
2	0	0	1	0
3	0	1	0	0
4	1	0	0	0

[a] Value of δ_{ik} at optimal solution.

Table 7
Optimal solution for *Model 2*

Stage	Alternative[a]				Input
	0	1	2	3	
1	1	0	0	0	43,136
2	1	0	0	0	30,195
3	1	0	0	0	27,779
4	1	0	0	0	22,222

[a] Value of δ_{ik} at optimal solution.

- Model 1 ：介紹與一般管理方法類似，都是試著最大化整體良率。
- Model 2：作者主要表達「經濟面」也是決策時的核心；因此以最大化利潤為目標。

結論

- 本論文提供兩個以經濟考量為基礎的數學模式，幫助管理者在品質管理上選擇較好的決策。
- 六標準差可以改善品質，也能獲得經濟面的利益，並且使得企業增加市場佔有率並且減少浪費成本。