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A simple approach for robust economic design of control charts

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Computers & Operations Research 34 (2007) 2001–2009

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Introduction

- 在設計管制圖時，必須決定樣本大小(n)、抽樣間隔頻率(h)、管制上下界限(k)。

- 經濟設計中，控制圖的參數選擇主要是製程控制成本的最小。而穩健經濟設計中的目標主要是減少發生金融上的損失。

Notation

b	fixed cost per sample
c	cost per unit sampled
d	mean time to search and fix the process
g	mean time to test and interpret the result per sample unit
p	power of the chart
T	quality control cycle time
M	hourly penalty cost for operating in out of control state
W	cost to locate and repair an assignable cause
Y	cost per false alarm
δ	process shift parameter
λ	process failure rate
α	probability of type-I error
$\Phi(.)$	distribution function of standard normal variate

Duncan's loss-cost function

$$L = \frac{b + cn}{h} + \frac{\lambda MB + \alpha Y / h + \lambda W}{1 + \lambda B},$$

where

$$B = ah + gn + d,$$

$$a = 1/p - 0.5 - \lambda h/12,$$

$$\alpha = 2\Phi(-k),$$

$$p = \Phi(-\delta\sqrt{n} - k) + \Phi(\delta\sqrt{n} - k).$$

Robust economic design for a process with multiple scenarios

The robust economic designs reported in the literature fall into two categories：

- Designs that consider the uncertainty in the estimation of the cost and process parameters.
- Designs that consider the discrete multiple scenarios of the process.

Linderman and Choo considered three different scenarios of the process with three process shifts and the corresponding out-of-control costs, while keeping all other arameters as constants. Three different designs have been suggested based on the following discrete optimization measures given below:

- 1. Absolute robustness.
- 2. Robust deviation.
- 3. Relative robustness.

Considering the process with j scenarios and the frequency of jth scenario as f_j, the weight of jth scenario can be calculated as

$$w_j = \frac{f_j}{\sum_j f_j}. \tag{2}$$

Since the failure rate of the process provides the information on the frequency of failures of the process per unit time (expressed as failures/hour), the relative frequency of occurrence of the failures for each scenario is given by its corresponding failure rate. Hence, Eq. (2) can be written as

$$w_j = \frac{\lambda_j}{\sum_j \lambda_j}. \tag{3}$$

Now the weighted expected cost of the process with j scenarios is

$$\eta = \sum_j w_j E(L_j/n, h, k) \tag{4}$$

where

$$E(L_j/n, h, k) = \frac{\lambda_j M_j B_j + \alpha Y_j/h + \lambda_j W_j}{1 + \lambda_j B_j} + \frac{b+c}{n}.$$

$$B_j = a_j h + gn + d_j,$$

$$a_j = 1/p_j - 0.5 - \lambda_j h/12,$$

$$p_j = \Phi(-\delta_j \sqrt{n} - k) + \Phi(\delta_j \sqrt{n} - k).$$

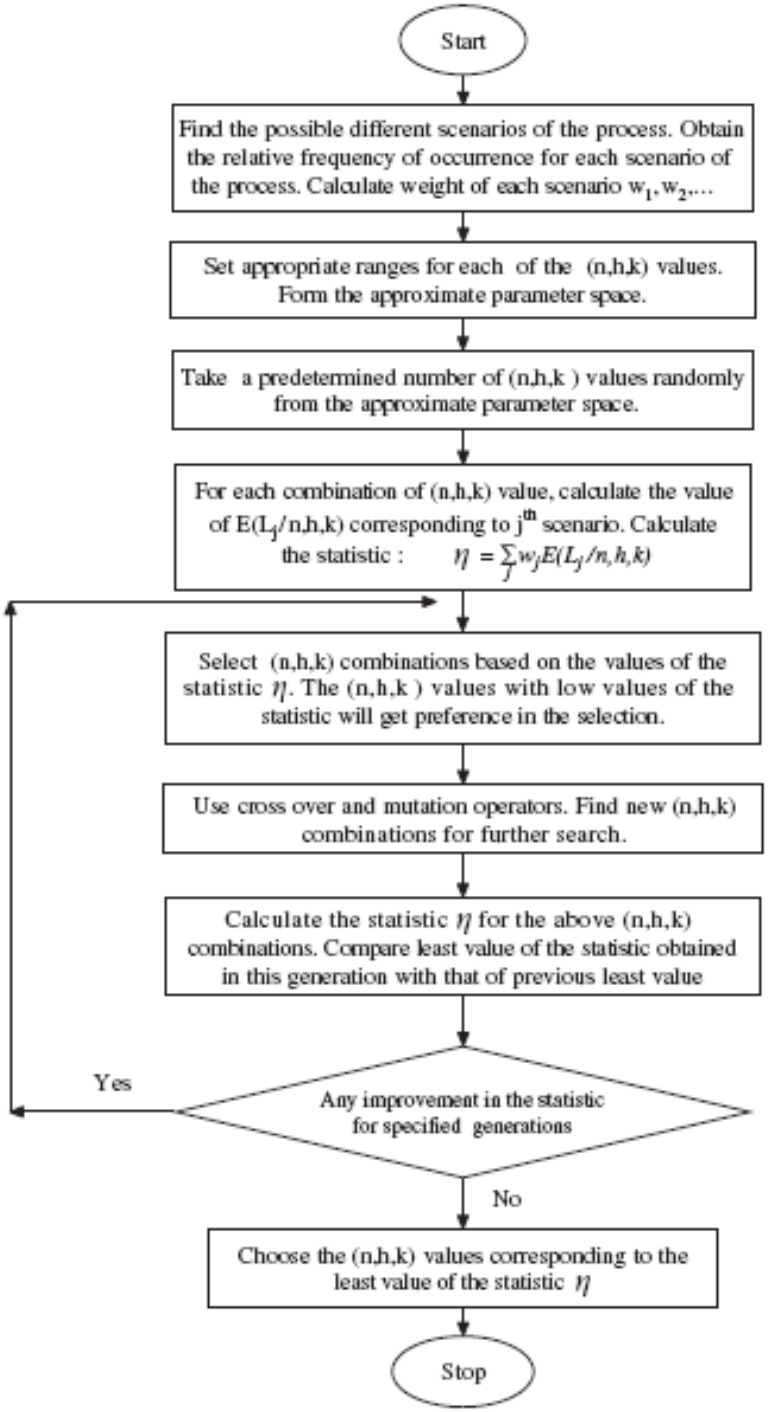
Start
Find the possible different scenarios of the process. Obtain the relative frequency of occurrence for each scenario of the process. Calculate weight of each scenario $w_1, w_2, \ldots$
Set appropriate ranges for each of the (n,h,k) values. Form the approximate parameter space.
Take a predetermined number of (n,h,k) values randomly from the approximate parameter space.
For each combination of (n,h,k) value, calculate the value of $E(L_j/n,h,k)$ corresponding to j^{th} scenario. Calculate the statistic : $\eta = \sum_j w_j E(L_j/n,h,k)$
Select (n,h,k) combinations based on the values of the statistic η. The (n,h,k) values with low values of the statistic will get preference in the selection.
Use cross over and mutation operators. Find new (n,h,k) combinations for further search.
Calculate the statistic η for the above (n,h,k) combinations. Compare least value of the statistic obtained in this generation with that of previous least value
Any improvement in the statistic for specified generations
Yes
No
Choose the (n,h,k) values corresponding to the least value of the statistic η
Stop

Table 1
Cost and process parameters at different scenarios

Scenario	b	c	W	Y	M	λ	δ	g	d
1	0.5	0.1	25	50	90	0.01	0.5	0.05	4
2	0.5	0.1	25	50	190	0.01	1.0	0.05	4
3	0.5	0.1	25	50	290	0.01	2.0	0.05	4

Table 2
Control chart parameter ranges in SGA

Parameter	Range	Size(bits)
n	2–65	6
h	0.5–40.0	12
k	1.0–5.0	9

Table 3
Parameters used in SGA based search procedure

Description of the SGA parameter	Magnitude/method
Length of chromosome (l)	27
Population size	200
Fitness function	Linear ranking
Selection	Stochastic universal sampling
Cross over probability (p_c)	0.7
Mutation probability (p_m)	$1/l$
Elitist strategy	True
Generation gap	0.9
Number of generations	50 without improvement

Table 4
Comparison of the results of robust designs with respect to new statistic

Criteria	n	h	k	$\sum w_i E(L_i/n,h,k)$
Absolute robustness	5	1.0398	2.5552	14.4038
Robust deviation	9	1.2938	2.2998	12.7896
Relative robustness	10	1.3086	2.3233	12.7482
Weighted expectation	10	1.1076	2.3699	12.7116

Table 5
Statistics of best objective values

Number of trials	300
Mimimum of best objective values	12.7116
Maximum of best objective values	12.7330
Average of best objective values	12.7118
Standard deviation of best objective values	0.0018

Table 6
Results of tests of significance

Criterion	Z-value	Inference
Absolute robustness	−1.6281e+004	Significant ($\ll -2.33$)
Robust deviation	−748.6308	Significant ($\ll -2.33$)
Relative robustness	−350.2592	Significant ($\ll -2.33$)

Summary and conclusions

- 穩健經濟設計可透過權重期望值的觀點，選擇控制圖中的參數。

- 三佰次的試驗中，此控制參數透過權重期望值及SGA的研究方法，比起其它的準則，是可提供較佳的solutions。

感想

- 讀完這篇PAPER後，作者運用簡單的統計方法，亦即frequency的簡單概念，來提升資序訊代表性，讓我覺得研究，其實也可以不利用很複雜的概念，而是可以回歸到一個簡單原理上去探討。

報告完畢

謝謝大家