Optimal burn-in time and warranty length under fully renewing combination free replacement and pro-rata warranty

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Keywords

- Burn-in
- Warranty
- Cost
- Failure time

Abstract

- 保固是產品銷售的一個重要因子，但常成為賣方的額外成本，而這些都取決於產品的可靠度。預燒是針對一產品製程外加一應力，以加速產品損壞，縮短產品壽命時間來增進產品靠度。
- 此篇研究在發展一個在產品免費換新結合FRW/PRW策略下的最佳預燒時間與保固長度。
- w　：產品保固長度
  w'　：可更換產品的保固時間長度
  [w,w']　：買方按比例付額維修的保固期
- 利用數值例子得出產品免費換新結合FRW/PRW對賣方來說是為保固最適策略。

Introduction

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- Replacement warranty policy：賣方需要維修且換新產品，買方不須出錢
- Pro-rata warranty policy：買方需要按保固長度的剩餘時間依比例付費維修或換新產品

結合兩者可使買賣雙方達到最適成本控制。

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- The fully renewing combination free replacement and pro-rata warranty police (FRW/PRW)：

w　　：產品保固期長度

w’　　：提早損壞時間長度

[w,w’]：按比例付費置換產品

Literature review

- (1) Nguyen and Murthy： 產品在最初期有較高的失效率，預燒可以減少保固成本的支出。
- (2) Kar and Nachlas： 提出一保固與預燒模型以檢驗產品利潤的管理，最佳預燒時間與最小化成本。
- (3) Yun et al ：最佳預燒時間與總平均成本最小化，此為製造的成本、預燒成本、累積保固成本的總和(under FRW)。
- (4) Sheu and Chien：在保固之下對可維修產品提出一般化預燒與實際作業模型。

- 成本模型：

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- 製造成本：

b：產品預燒時間

T：產品失效時間的隨機變數

f(t)：失效時間機率密度函數

F(t)：失效時間累積函數 t≧0

c_0 ：製造成本 c_1：固定預燒成本

c_2 ：每單位時間預燒成本

$c_1 + c_2$ t， t≦b 失效

$c_1 + c_2$ t， t≦b 未失效

E[B(t)]：每單位預燒成本期望值1-F(b)：產品在預燒後留存機率

$$\begin{aligned}E[B(t)] &= \int_0^b (C_1 + C_2 t) f(t)\,\mathrm{d}t + \int_b^\infty (C_1 + C_2 b) f(t)\,\mathrm{d}t \\ &= C_1 + C_2 \left[\int_0^b t f(t)\,\mathrm{d}t + b\int_b^\infty f(t)\,\mathrm{d}t\right] \\ &= C_1 + C_2 \left[-t(1 - F(t))|_0^b + \int_0^b (1 - F(t))\,\mathrm{d}t \right. \\ &\quad \left. + b\left(1 - \int_0^b f(t)\,\mathrm{d}t\right)\right] = C_1 + C_2 \int_0^b (1 - F(t))\,\mathrm{d}t, \end{aligned} \tag{1}$$

M(b)：預燒時間為b的期望製造成本

$$M(b) = \frac{C_0 + C_1 + C_2 \int_0^b (1 - F(t))\,\mathrm{d}t}{1 - F(b)}, \tag{2}$$

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- 保固成本：Wald’s renewal equation

(1)期望保固成本：E[W(w,b)]

(2)每單位置換的期望保固成本：E[v(t)]

(3)期望更換次數：E[N(w)] ,in interval [0,w]

$$E[W(w,b)] = E[N(w)]E[v(t)].$$

- 期望維修數

假設每一件產品服從同一分配且累積機率函數為F(w)；期望更換數N(w)須滿足保固條件而有幾何分配，即直到壞掉為止,其機率分配為

$P[N(w)=n] = F(w)^n\,[1-F(w)]$，n=0或1

所以更換數為 $$E[N(w)] = \frac{F(w)}{1-F(w)}.$$

- 修正 Blischke and Murchy
  Mitra and Patankar 的成本模型

則在FRW/PRW策略下每單位更換保固成本函數

$$v(t)=\begin{cases} C_3, & 0\leqslant t<w', \\ kC_3\left(1-\delta\frac{t-w'}{w-w'}\right), & w'\leqslant t<w, \\ 0, & \text{otherwise}, \end{cases}$$

C_3：每單位失效置換成本在保固長度下

K ：C_3的比例係數　　　　0≦K≦1

δ ：在保固期失效時間的比例係數　0≦δ≦1

- 在預燒後的失效機率與累積函數分別為 $f_b(t)$ 與 $F_b(t)$

每單位更換的期望保固成本為

$$
\begin{aligned}
E[v(t)] &= \int_0^{w'} C_3 f_b(t)\,\mathrm{d}t + \int_{w'}^{w} kC_3\left(1-\delta\frac{t-w'}{w-w'}\right) f_b(t)\,\mathrm{d}t \\
&= C_3F_b(w') + \int_{w'}^{w} kC_3 f_b(t)\,\mathrm{d}t - kC_3\delta \\
&\quad \times \int_{w'}^{w}\left(\frac{t-w'}{w-w'}\right) f_b(t)\,\mathrm{d}t = C_3\frac{F(b+w')-F(b)}{1-F(b)} \\
&\quad + kC_3\left[\frac{F(b+w)-F(b+w')}{1-F(b)}\right] \\
&\quad - kC_3\delta\left[\frac{F(b+w)-F(b)}{1-F(b)}\right. \\
&\quad \left.-\frac{(F(w+b)-F(w'+b))(F(w+b)+F(w'+b)-2F(b))}{(w-w')(1-2F(b)-F^{(2)}(b))}\right].
\end{aligned}
$$

- 在預燒時間b，保固長度w之下期望保固成本：

$$
\begin{aligned}
&E[W(w,b)] \\
&= E[N(w)]E[v(t)] \\
&= \frac{F(w)}{1-F(w)}\left\{ C_3 \frac{F(b+w')-F(b)}{1-F(b)} \right. \\
&\quad + kC_3 \left\{ \left[\frac{F(b+w)-F(b+w')}{1-F(b)} \right] \right. \\
&\quad - \delta \left[\frac{F(b+w)-F(b)}{1-F(b)} \right. \\
&\quad \left.\left.\left. - \frac{(F(w+b)-F(w'+b))(F(w+b)+F(w'+b)-2F(b))}{(w-w')(1-2F(b)-F^{(2)}(b))} \right] \right\} \right\}.
\end{aligned}
$$

- 3/3
- 總平均成本：製造成本與保固成本總和

$$
\begin{aligned}
C(w,b) &= M(b) + E[W(w,b)] \\
&= \frac{C_0 + C_1 + C_2(1-F(b))^2}{1-F(b)} \\
&\quad + C_3\frac{F(b+w')-F(b)}{1-F(b)} \\
&\quad + kC_3\left\{\left[\frac{F(b+w)-F(b+w')}{1-F(b)}\right] - \delta\left[\frac{F(b+w)-F(b)}{1-F(b)}\right.\right. \\
&\quad \left.\left. - \frac{(F(w+b)-F(w'+b))(F(w+b)+F(w'+b)-2F(b))}{(w-w')(1-2F(b)-F^{(2)}(b))}\right]\right\}.
\end{aligned}
$$

Numerical examples

- 假設失效時間服從指數與韋伯分配，並以四種保固策略做梯度搜尋分析

(1) FRW/PRW k=0.5 δ =0.5

典型免費置換與按比例保固策略

(2) FRW/LSW k=0.5 δ =0

全額退款保固策略

(3) FRW/PRW k=1 δ =1

比例退款保固策略

(4) fully free renewing k=1 δ =0

完全免費更新策略

1/2 指數分配型

- 指數累積機率分配函數

$$F(t) = p(1 - e^{-\lambda_1 t}) + (1 - p)(1 - e^{-\lambda_2 t})$$

$$\text{for} \quad t \geq 0, 0 < p < 1 \quad \text{and} \quad \lambda_1, \lambda_2 > 0.$$

- 參數設定為p=0.1 ， λ_1=4 ， λ_2 =0.08

  c_0 =5 ， c_1 =0.2 ， c_2 =5， c_3 =10

第(1)到第(3)策略的數據表如下

Table 1

Total mean cost for the mixed exponential distributed failure time example

b	Policy I					Policy II					Policy III				
	$w = 1.5$	$w = 1.75$	$w = 2.0$	$w = 2.25$	$w = 2.5$	$w = 1.5$	$w = 1.75$	$w = 2.0$	$w = 2.25$	$w = 2.5$	$w = 1.5$	$w = 1.75$	$w = 2.0$	$w = 2.25$	$w = 2.5$
0	10.9364	10.92474	10.92512	10.9302	10.9374	10.9707	10.99492	11.01854	11.04165	11.0643	11.09579	11.0006	10.95488	10.9289	10.91265
0.01	10.9355	10.92381	10.9242	10.9293	10.9366	10.9682	10.99251	11.01619	11.03935	11.062	11.10223	11.0069	10.96107	10.935	10.91884
0.02	10.9349	10.92319	10.92359	10.9287	10.936	10.9661	10.99049	11.01422	11.03743	11.0602	11.1088	11.0132	10.96736	10.9413	10.9251
0.03	10.9346	10.92288	10.92328	10.9284	10.9357	10.9644	10.98884	11.01261	11.03587	11.0587	11.11548	11.0197	10.97373	10.9477	10.93144
0.04	10.9347	10.92288	10.92325	10.9284	10.9357	10.9631	10.98753	11.01135	11.03465	11.0575	11.12227	11.0262	10.98017	10.9541	10.93783
0.05	10.9349	10.9231	10.92349	10.9286	10.936	10.9621	10.98654	11.0104	11.03375	11.0566	11.12916	11.0328	10.98667	10.9605	10.94427
0.06	10.9355	10.9236	10.92398	10.9291	10.9365	10.9614	10.98586	11.00976	11.03315	11.0561	11.13615	11.0395	10.99323	10.967	10.95074
0.07	10.9363	10.92433	10.9247	10.9298	10.9372	10.9609	10.98548	11.00941	11.03283	11.0558	11.14322	11.0463	10.99984	10.9736	10.95725
0.08	10.9373	10.9253	10.92564	10.9308	10.9382	10.9608	10.98536	11.00933	11.03279	11.0558	11.15037	11.0531	11.00648	10.9801	10.96378
0.09	10.9386	10.92647	10.92678	10.9319	10.9393	10.9609	10.98551	11.00951	11.033	11.056	11.1576	11.0599	11.01316	10.9867	10.97033
0.1	10.9401	10.92784	10.92813	10.9332	10.9407	10.9613	10.9859	11.00993	11.03345	11.0565	11.16489	11.0668	11.01987	10.9933	10.9769
0.2	10.964	10.95043	10.95013	10.955	10.9622	10.976	11.00071	11.0249	11.04858	11.0718	11.24088	11.1372	11.08762	11.0596	11.04227

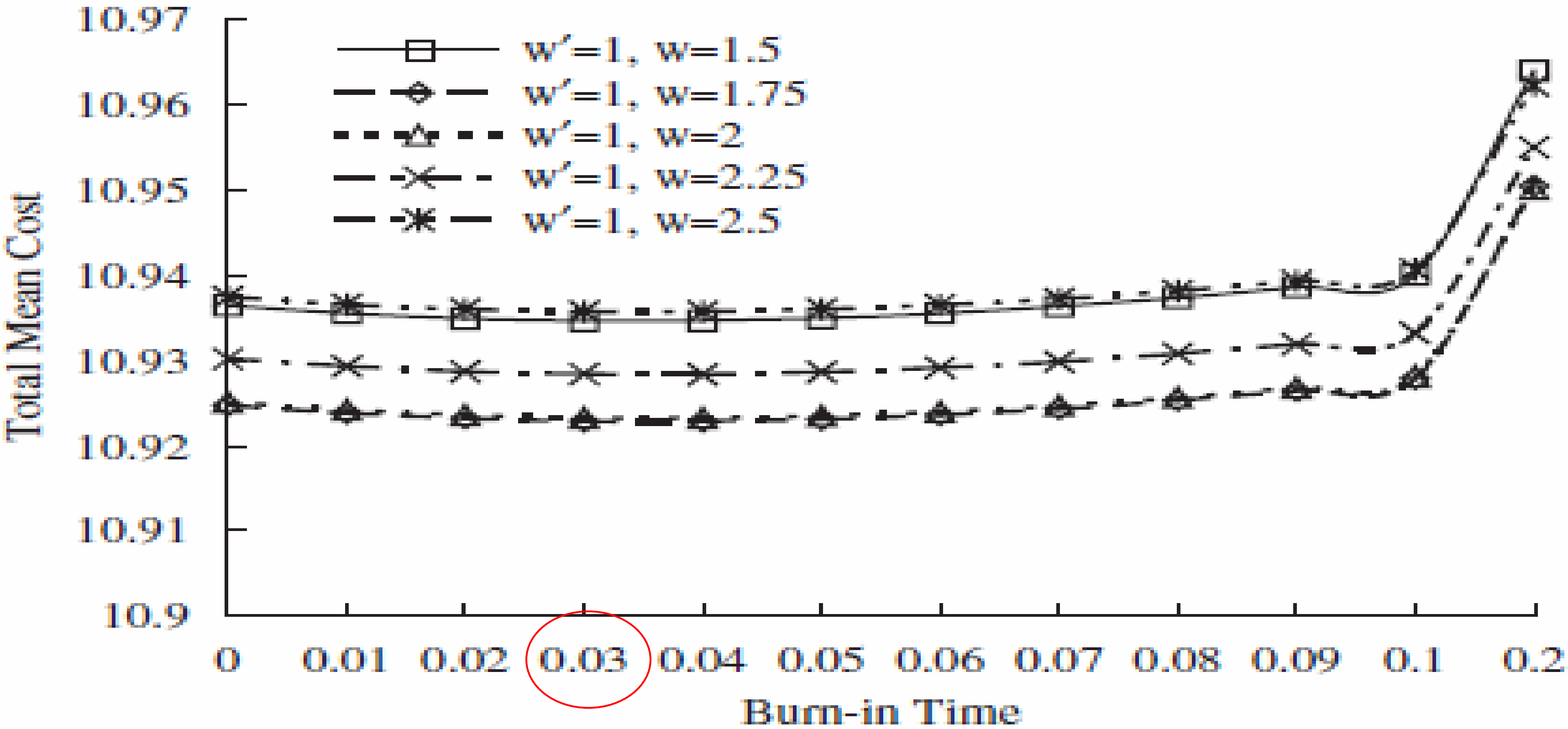

Fig. 2. Burn-in time and total mean cost of Policy I for the mixed exponential distribution.

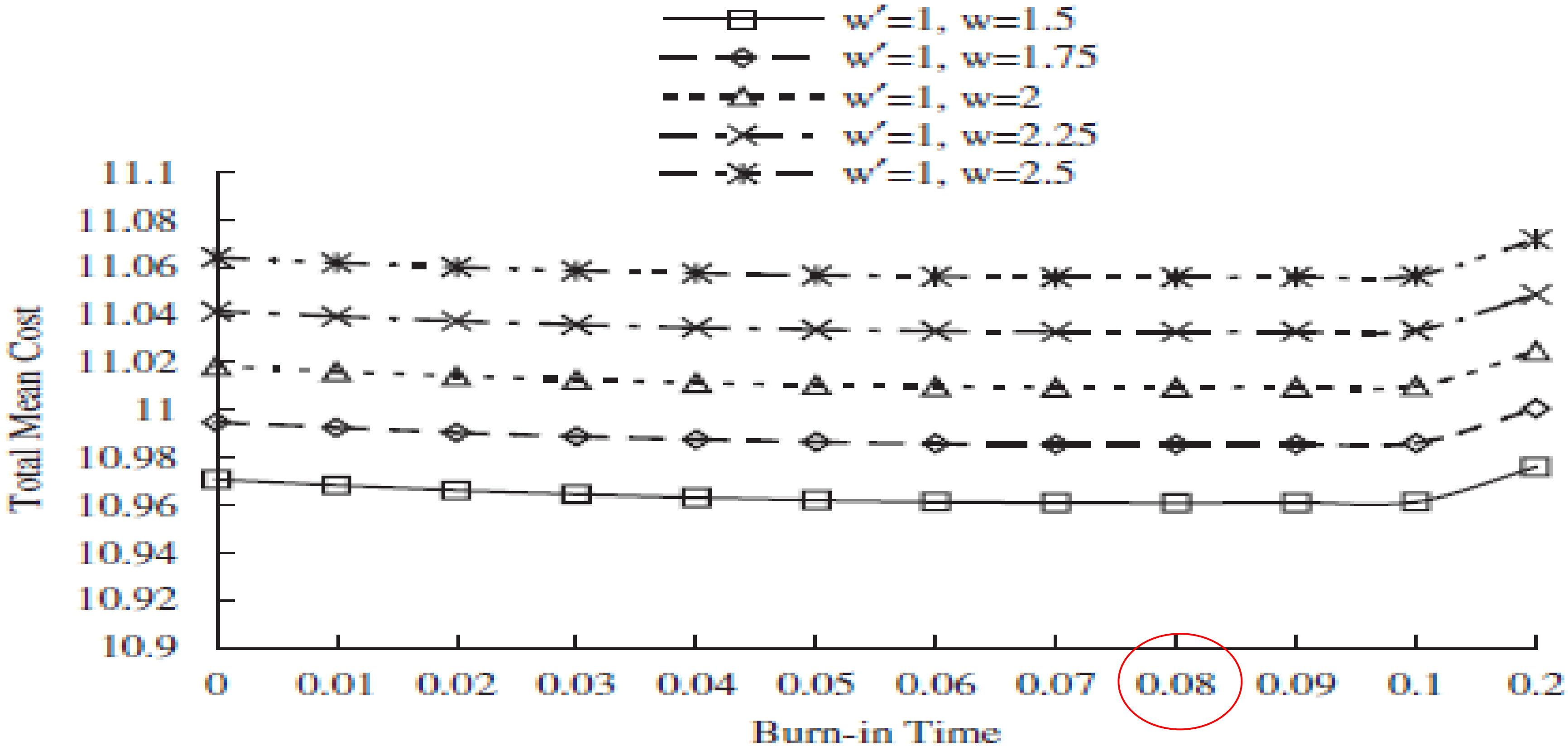

Fig. 3. Burn-in time and total mean cost of Policy II for the mixed exponential distribution.

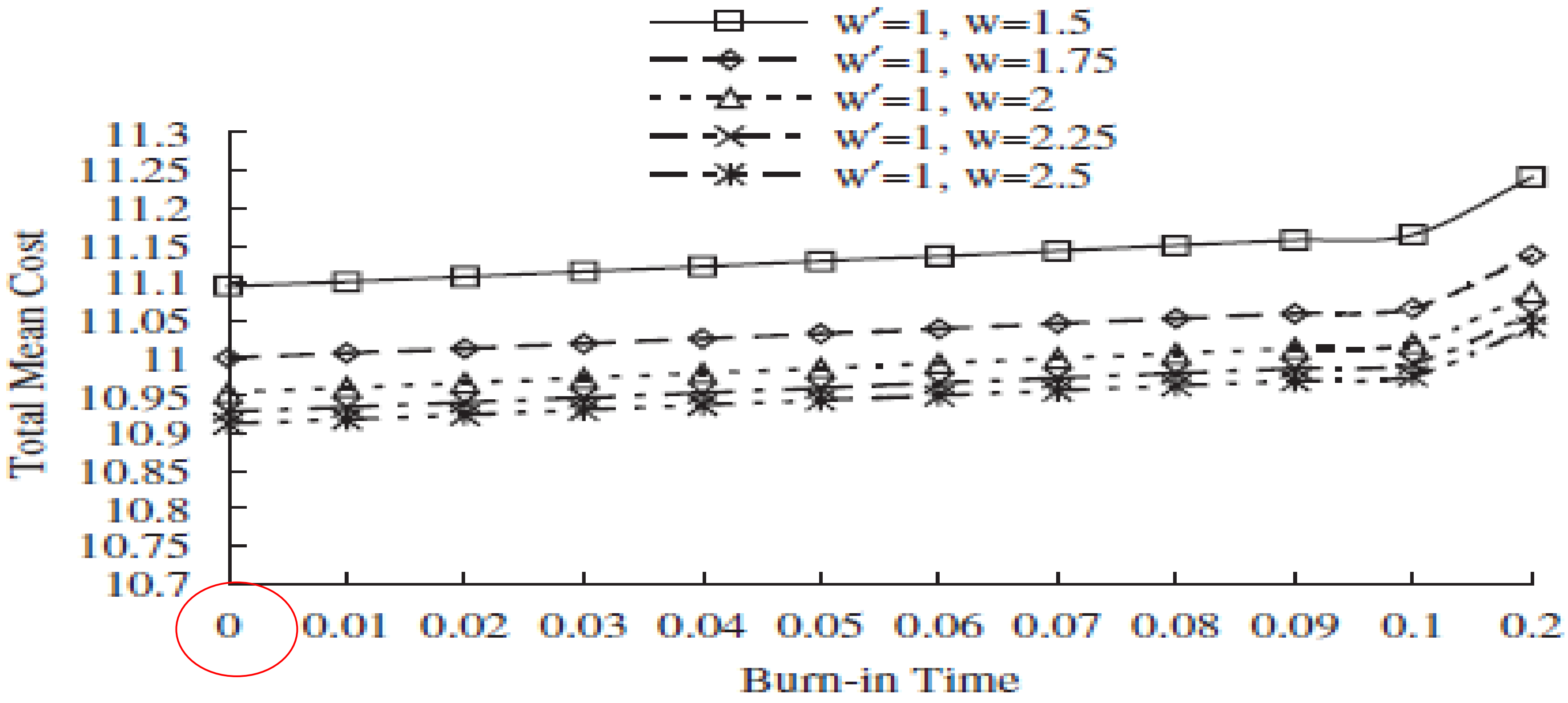

Fig. 4. Burn-in time and total mean cost of Policy III for the mixed exponential distribution.

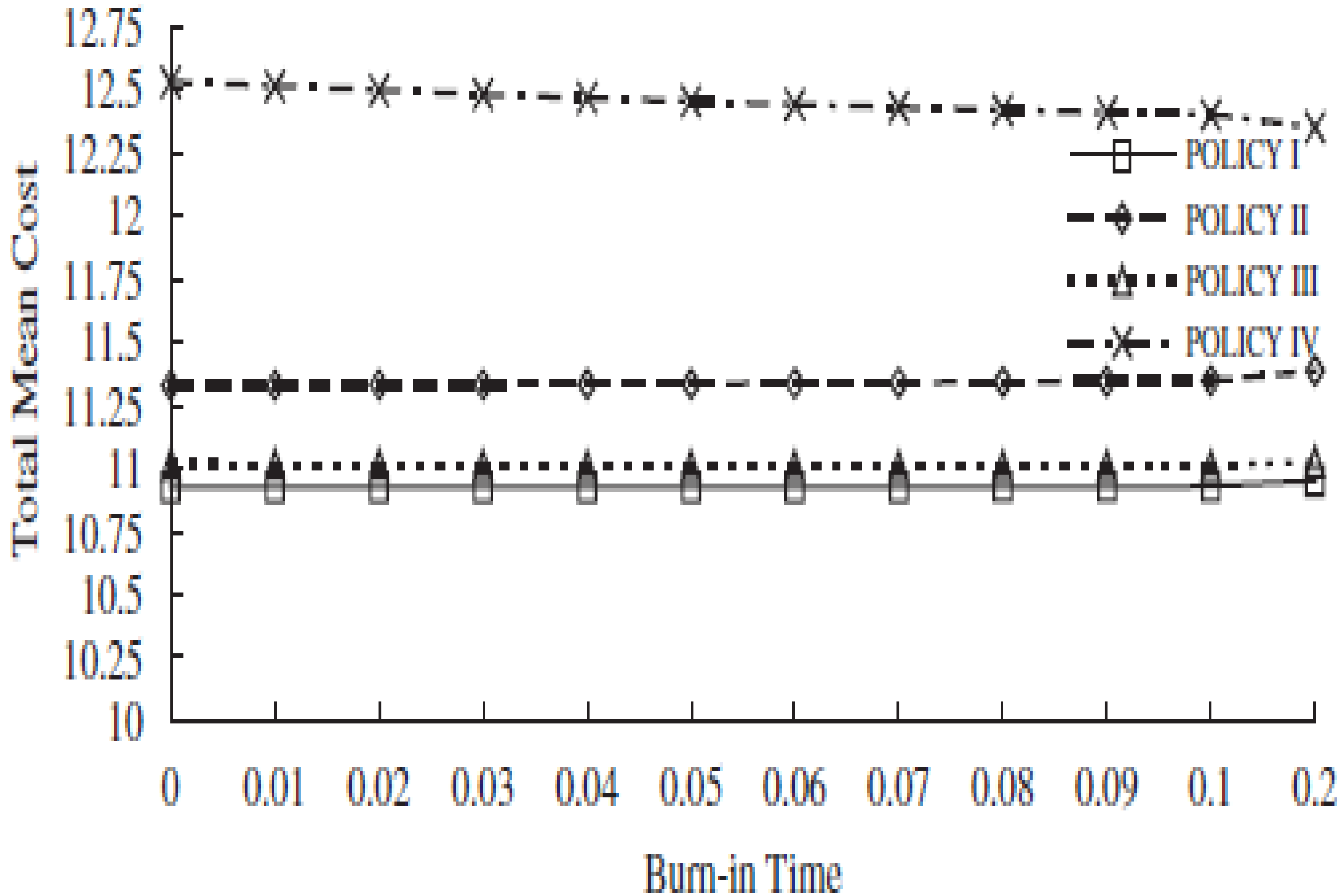

Fig. 5. Burn-in time versus total mean cost of various warranty policies for the mixed exponential distribution.

2/2 韋伯分配型

- 韋伯累積機率分配函數

$$F(t) = p(1 - e^{-(\lambda_1 t)^{\beta_1}}) + (1-p)(1 - e^{-(\lambda_2 t)^{\beta_2}})$$

$$\text{for} \quad t \geq 0,\ 0 < p < 1,\ 0 < \beta_1 < 1,\ \beta_2 < 1$$

$$\text{and} \quad \lambda_1, \lambda_2 > 0.$$

- P=0.1 λ_1=4 λ_2=0.08 β_1 =0.5 β_2 =3
  c_0=5 c_1 =0.2 c_2 =5 C_3 =10

第(1)到第(3)策略的數據表如下

Table 2

Total mean cost for mixed Weibull distributed failure time example

b	Policy I					Policy II					Policy III				
	$w = 1.5$	$w = 1.75$	$w = 2.0$	$w = 2.25$	$w = 2.5$	$w = 1.5$	$w = 1.75$	$w = 2.0$	$w = 2.25$	$w = 2.5$	$w = 1.5$	$w = 1.75$	$w = 2.0$	$w = 2.25$	$w = 2.5$
0	10.2735	10.2741	10.2748	10.2755	10.2762	10.2979	10.299	10.3005	10.3019	10.3033	10.2041	10.2040	10.2040	10.2041	10.2042
0.1	10.2594	10.2609	10.2608	10.2616	10.2623	10.2723	10.2737	10.2751	10.2765	10.2781	10.2245	10.2245	10.2245	10.2246	10.2247
0.2	10.2602	10.2609	10.2616	10.2624	10.2633	10.27	10.2715	10.273	10.2745	10.2762	10.2344	10.2344	10.2344	10.2345	10.2346
0.3	10.262	10.2627	10.2634	10.2643	10.2652	10.2702	10.2716	10.273	10.2746	10.2764	10.2416	10.2415	10.2416	10.2417	10.241
0.4	10.264	10.2647	10.2655	10.2663	10.2673	10.271	10.2724	10.2739	10.2756	10.2774	10.2473	10.2473	10.2473	10.2474	10.2476
0.5	10.266	10.2667	10.2675	10.2684	10.2694	10.2722	10.2736	10.2752	10.277	10.2789	10.252	10.252	10.2521	10.2522	10.2523
0.6	10.268	10.2687	10.2696	10.2705	10.2716	10.2736	10.2751	10.2767	10.2786	10.2806	10.2562	10.2561	10.2562	10.2563	10.2565
0.7	10.27	10.2707	10.2716	10.2726	10.2737	10.2752	10.2767	10.2784	10.2804	10.2825	10.2598	10.2598	10.2599	10.26	10.2602
0.8	10.2719	10.2727	10.2737	10.2747	10.2759	10.2769	10.2785	10.2803	10.2823	10.2846	10.2632	10.2632	10.2632	10.2634	10.2636
0.9	10.2739	10.2748	10.2757	10.2768	10.278	10.2787	10.2804	10.2823	10.2844	10.2867	10.2663	10.2663	10.2664	10.2665	10.2667
1	10.2759	10.2768	10.2778	10.2789	10.2803	10.2806	10.282	10.2843	10.2865	10.289	10.2693	10.2693	10.269	10.2695	10.2697
1.1	10.278	10.2789	10.2799	10.2811	10.2825	10.2826	10.2845	10.2865	10.2889	10.2915	10.27224	10.2721	10.2722	10.2724	10.2726
1.2	10.2801	10.281	10.2821	10.2834	10.2848	10.2848	10.2867	10.2889	10.291	10.294	10.275	10.275	10.2751	10.2752	10.2755
1.3	10.2822	10.2832	10.2844	10.2857	10.2872	10.287	10.289	10.2913	10.2939	10.2968	10.2778	10.2778	10.2778	10.278	10.2783
1.4	10.2844	10.2855	10.2867	10.2881	10.2897	10.2894	10.2915	10.2939	10.2966	10.2996	10.2806	10.2805	10.2806	10.2809	10.2812
1.5	10.2867	10.2878	10.2891	10.2906	10.2922	10.2919	10.2941	10.2966	10.2994	10.3026	10.2835	10.2834	10.2835	10.2837	10.284

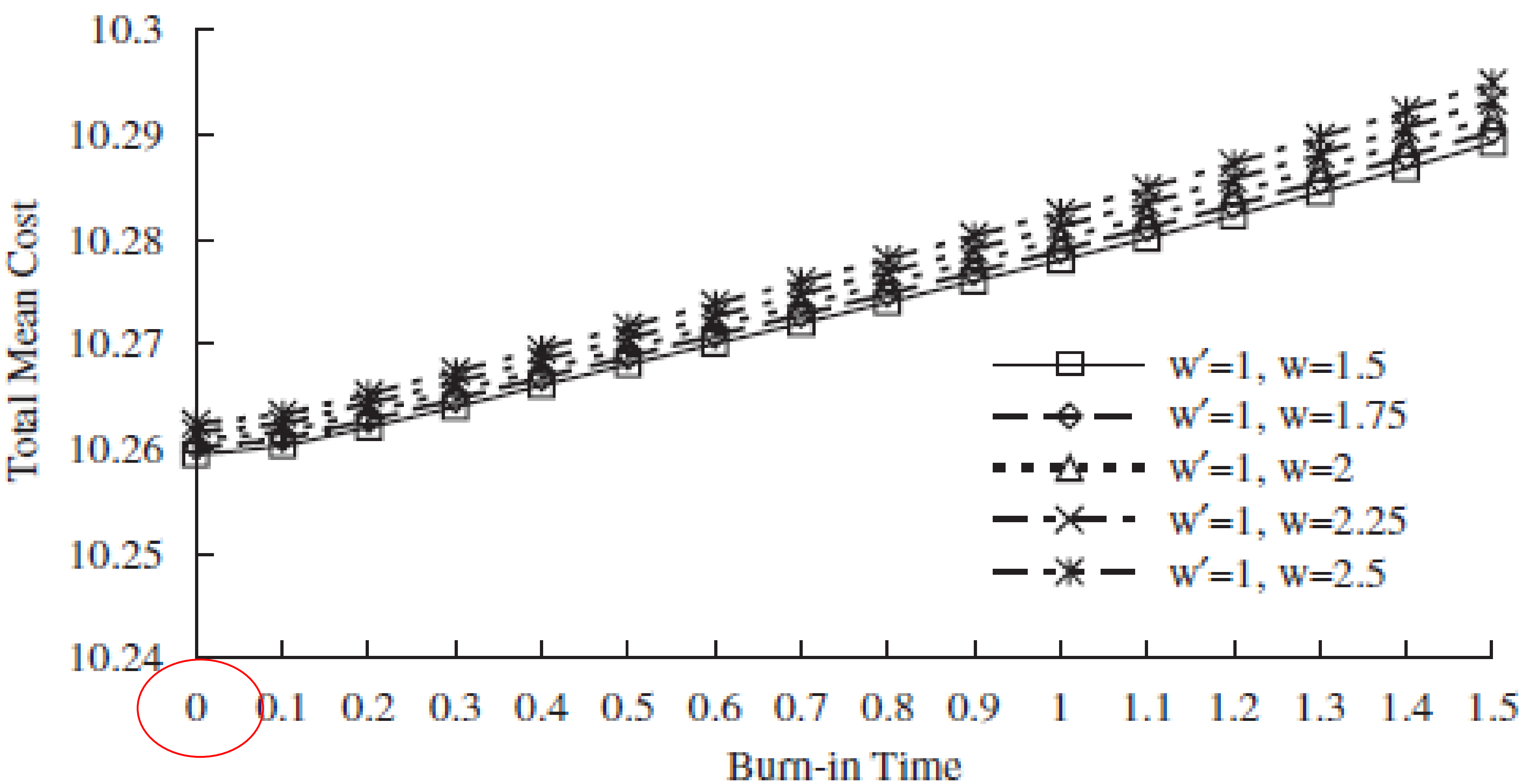

Fig. 6. Burn-in time and total mean cost of Policy I for the mixed Weibull distribution.

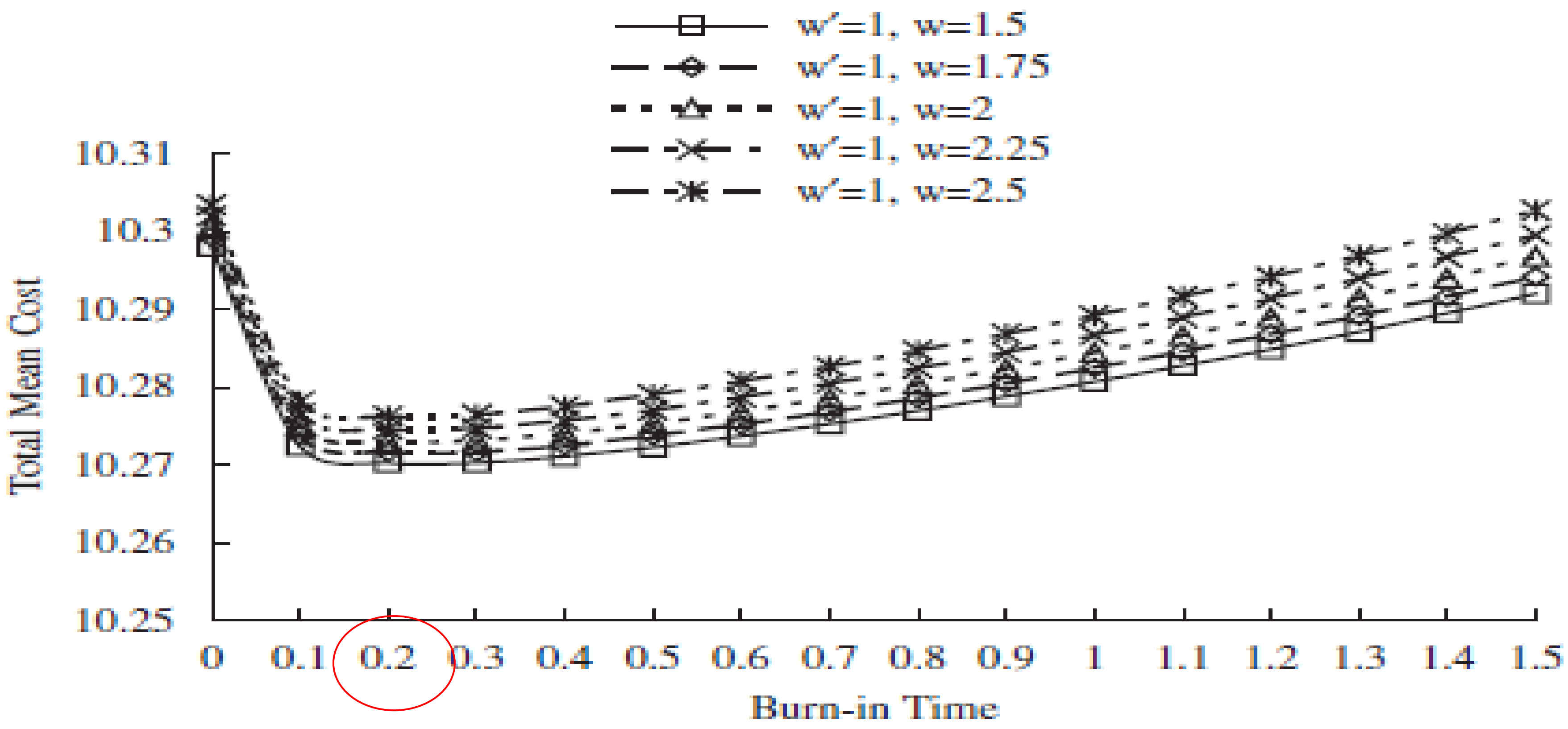

Fig. 7. Burn-in time and total mean cost of Policy II for the mixed Weibull distribution.

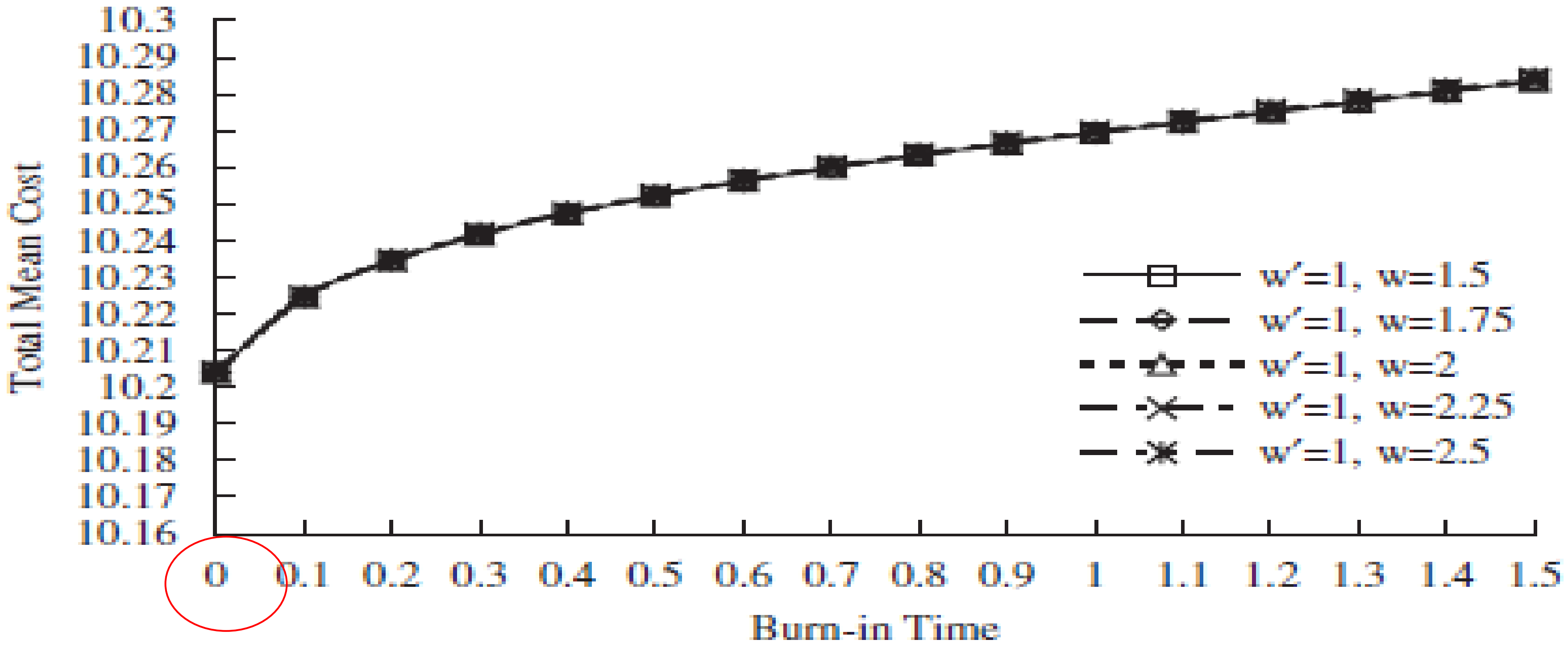

Fig. 8. Burn-in time and total mean cost of Policy III for the mixed Weibull distribution.

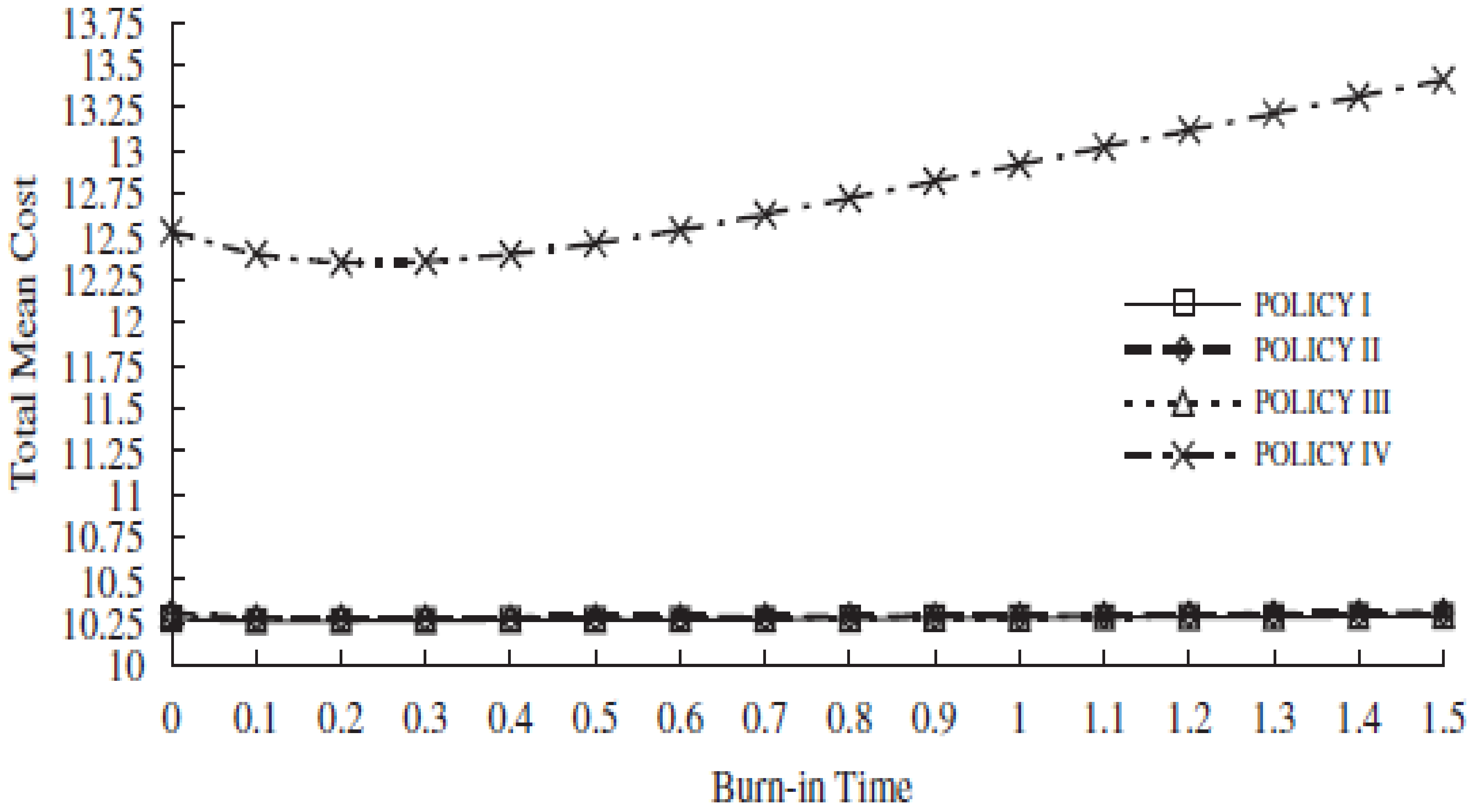

Fig. 9. Burn-in time versus total mean cost of various warranty policies for the mixed Weibull distribution.

Conclusions

- (1)按比例退款保固策略因為成本過高，不建議執行預燒
- (2)指數型的典型與總額退款策略小於0.1年最佳 w'/w 不可太大或太小
- (3)韋伯型的典型與總額退款策略介於0.1到0.2 w'/w 不可太大
- (4)全免費更新保固策略成本最高

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