Optimal periodic preventive maintenance policy for leased equipment

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Introduction(1/2)

- 1990年後有越來越多的企業開始朝向租賃設備而不是購買設備，主要的原因有兩個，
  - 科技技術的進步
  - 購買的成本太高

由於上述兩項原因，所以租賃設備行業的快速興起

Introduction(2/2)

Nisbet and Ward [2001]，在決定放射療法設備為採購或租賃時，提到有關租賃為發生時相關要素

- Lessor
- Lessee
- Lease contract

- Jaturonnatee et al.[2006]探討租賃期間執行k次預防維護，設備經第i次預防維護後的失效率會縮減δ_j, 透k、t、δ參數的變動，來決定罰金與預防維護成本之間互抵效應最化的問題

  - 缺點
    - 最佳化的問題：包括2k+1參數如何選擇最佳
    - 在兩個預防維護行動的時間間隔一般是未限制的，意謂著從實際執行的角度來說，是不實用的

Model formulation(1/3)

Lease contract

- Y≦ τ 不用罰金，Y> τ 其罰金成本為(Y- τ)C
  - Y:實際修復時間
  - τ ：出租人合約上定的修復時間
- 設備失效就付罰金

Model formulation(2/3)

- Modeling failures and PM actions

The intensity function for the occurrence of failures with periodic PM actions is given by

$$\lambda(t) = \lambda_0(t) - \sum_{i=0}^{j} \delta_i \quad \text{for } t_j \le t < t_{j+1} \tag{2}$$

where $t_0 = 0$ and $\delta_0 = 0$.

Model formulation(3/3)

- Modeling costs

$$TC_f = \sum_{i=1}^{N(t)} \tilde{C}_i$$

維修失效的成本

$$TC_p = \sum_{j=1}^{k} (a + b\delta_j)$$

預防維護成本=固定成本+變動成本

$$\phi_1[N(L), Y_i, \tau] = C_t \left[\sum_{i=1}^{N(L)} \max(0, Y_i - \tau) \right]$$

Penalty 1 cost

$$\phi_2(N(L)) = C_n N(L).$$

Penalty 2 cost

Model analysis(1/5)

- Expected number of failures

沒預防維護

$$E[N(L)] = \Lambda_0(L) = \int_0^L \lambda_0(t)\mathrm{d}t$$

有定期預防維護

$$E[N(L)] = \Lambda(L) = \Lambda_0(L) - \sum_{j=1}^{k} \delta_j(L - t_j).$$

Model analysis(2/5)

- Expected costs → $E(\mathrm{TC}_f) = C_f \Lambda(L)$

租期中失效的
預期次數

$$E[\phi_1(N(L), Y_i, \tau)] = C_t \Lambda(L) \left\{ \int_{\tau}^{\infty} (y-\tau) g(y) \mathrm{d}y \right\}$$

$$= C_t \Lambda(L) \int_{\tau}^{\infty} [1 - G(y)] \mathrm{d}y$$

the total expected
Penalty 1 cost

$$E[\phi_2(N(L))] = C_n \Lambda(L)$$

the total expected
Penalty 2 cost

Model analysis(3/5)

- Total expected cost to the lessor

$$J(T,\underset{\sim}{\delta}) = C_f \Lambda(L) + \sum_{j=1}^{k}(a + b\delta_j) + C_t \Lambda(L)\int_{\tau}^{\infty}[1 - G(y)]\mathrm{d}y + C_n \Lambda(L) \quad (13)$$

where $\underset{\sim}{\delta} = \{\delta_1, \delta_2, \ldots, \delta_{k(T)}\}$. Define

$$C' = C_f + C_t \int_{\tau}^{\infty}[1 - G(y)]\mathrm{d}y + C_n.$$

Then, (13) can be rewritten as

$$J(T,\underset{\sim}{\delta}) = C'\left[\Lambda_0(L) - \sum_{j=1}^{k}\delta_j(L - t_j)\right] + \sum_{j=1}^{k}(a + b\delta_j) \quad (14)$$

Model analysis(4/5)

- Optimization

Stage 1: For a fixed T, $k(T)$ satisfies $k(T)T < L \leq (k(T)+1)L$ and from (14) we have

$$J_1(\underset{\sim}{\delta}; T) = C'\Lambda_0(L) + ak - C'\sum_{j=1}^{k(T)} \delta_j\left(L - jT - \frac{b}{C'}\right) \tag{15}$$

subject to the constraints

$$0 \leq \delta_j(T) \leq \lambda_0(jT) - \sum_{i=1}^{j-1} \delta_i^*(T), \quad 1 \leq j \leq k(T). \tag{16}$$

Stage1中，藉由固定T，由最小化成本函數過程，求得 $\underset{\sim}{\delta}^*(T)$

Model analysis(5/5)

- Optimization

Step 1: $k=1$.
Step 2: Evaluate $T_{\min}=L/(k+1)$ and $T_{\max}=L/k$.
Step 3: Find $T^*(k)$ which yields a minimum for $J(T,\underline{\delta}^*(T))$ over the interval $T_{\min}<t\le T_{\max}$.
Step 4: $k\leftarrow k+1$. If $k<\bar{k}=\lceil C'\Lambda(L)/a\rceil$ then go to Step 3; else, go to Step 5.

Step 5: Search $J(T^*(k),\underline{\delta}^*(T^*(k)))$ for $1<k<\bar{k}$ to determine k^* which yields the smallest value for $J(T^*(k),\underline{\delta}^*(T^*(k)))$. T^* is given by $T^*=T^*(k^*)$. Using this, the optimal PM actions are given by $\underline{\delta}^*=\underline{\delta}^*(T^*)$ and the minimum expected cost to the lessor given by $J(T^*,\underline{\delta}^*(T^*))$.

由satge1得到 $\underline{\delta}^*(T)$ ，再藉由上述計算步驟，可得到一最佳的預防維護週期時間

Numerical example(1/5)

假設失效機率分配為韋伯分配

$L = 5$ (years), $C_f = 100$ (\$), $C_t = 300$ (\$),

$C_n = 200$ (\$), $a = 100$ (\$), $b = 50$ (\$),

$\tau = 2$ (days), $m = 0.5$, and $\varphi = 0.5$.

We consider two values of β (2 and 3).

Numerical example(2/5)

- No penalty costs

Table 1
$T^*(k)$ and $J(T^*(k), \underline{\delta}^*(T^*(k)))$ versus k

k	$\beta=2$		$\beta=3$	
	$T^*(k)$	$J(T^*(k), \underline{\delta}^*(T^*(k)))$	$T^*(k)$	$J(T^*(k), \underline{\delta}^*(T^*(k)))$
1	2.5000	$1600.00	3.0000	$8550.00
2	1.6667	$1366.67	1.7095	$7410.33
3	1.2500	**$1300.00**	1.2500	$6706.25
4	1.0000	$1300.00	1.0000	$6300.00
5	0.8333	$1333.33	0.8333	$6055.56
6	0.7143	$1385.71	0.7143	$5906.12
7	0.6250	$1450.00	0.6250	$5817.19
8	0.5556	$1522.22	0.5556	$5769.14
9	0.5000	$1600.00	0.5000	**$5750.00**
10	0.4545	$1681.82	0.4545	$5752.07

Note that $k^*=3$ for $\beta=2$ and $k^*=9$ for $\beta=3$.

Numerical example(3/5)

- With penalty costs

Optimal parameters of PM policy [$\beta = 2$]

Case	T^*	δ_1^*	$J^* = J(T^*, \delta^*)$
No penalty	1.2500	2.5000	$1300.00
Penalty 1	0.7143	1.4286	$1820.70
Penalty 2	0.6250	1.2500	$2075.00
Penalties 1 and 2	0.5000	1.0000	$2404.50

Table 3
Optimal parameters of PM policy [$\beta = 3$]

Case	T^*	k^*	$J^* = J(T^*, \delta^*)$
No penalty	0.5000	9	$5750.00
Penalty 1	0.2778	17	$7312.50
Penalty 2	0.2273	21	$8034.90
Penalties 1 and 2	0.1923	25	$8969.90

Numerical example(4/5)

- Sensitivity analysis

Parameters		τ (days)							
b ($)	C_n ($)	1		2		3		∝	
		T^*	J^* ($)	T^*	J^* ($)	T^*	J^* ($)	T^*	J^* ($)
20	0	0.2273	5784.02	0.2632	5296.66	0.2778	4973.76	0.4167	3879.51
	100	0.1923	6564.16	0.2083	6156.74	0.2273	5895.36	0.2778	5082.72
	200	0.1724	7241.04.	0.1852	6883.94	0.1923	6658.60	0.2174	5982.61
	300	0.1563	7848.01	0.1667	7526.67	0.1724	7325.82	0.1852	6734.43
50	0	0.2381	7827.21	0.2778	7312.50	0.3125	6968.53	0.5000	5750.00
	100	0.2000	8638.97	0.2174	8216.34	0.2381	7943.77	0.2941	7084.43
	200	0.1786	9336.99	0.1923	8969.90	0.2000	8737.14	0.2273	8034.90
	300	0.1613	9958.84	0.1667	9629.17	0.1724	9142.32	0.1923	8814.72
80	0	0.2500	9860.09	0.2941	9313.22	0.333	8940.86	0.6250	7539.84
	100	0.2083	10,707.71	0.2273	10,267.99	0.2500	9982.38	0.3125	9068.36
	200	0.1852	11,426.06	0.2000	11,050.91	0.2083	10,809.91	0.2381	10,078.23
	300	0.1613	12,066.02	0.1724	11,728.26	0.1786	11,516.92	0.2000	10,889.60

Numerical example(5/5)

- Comparison between policies 1 and 2

Case	Policy 1		Policy 2			Increase in J^* (%)
	k^*	J^*	T^*	k^*	J^*	
No penalty	10	$5437.03	0.5000	9	$5750.00	3.13
Penalty 1	19	$7712.87	0.2778	17	$7312.50	2.17
Penalty 2	20	$7712.87	0.2273	21	$8034.90	3.22
Penalties 1 and 2	26	$8610.66	0.1923	25	$8969.90	0.05

Conclusion

- 在定期預防維護的策略下，為租方建置一個最小的總預期成本項目
- 提供一個新的策略跟Jaturonnatee et al.[2006]所建議的策略做比較

未來成本模型的延伸方向

- 隨著設備年紀的增加小維修成本也隨著增加
- 不同的合約項目