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Optimal preventive maintenance policy for leased equipment using failure rate reduction

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Introduction

- 很多的企業在為顧客提供一服務或產品時，需要透過多種設備才有辦法滿足顧客的需求，由於技術的創新增加了設備的複雜性，因此可能需要花許多的成本去維護這些設備，所以很多企業有趨向於租賃設備而不選擇購買設備。

Mathematical formulation

L(設備租賃期間)內執行n次預防維護，假設備每次維護後 δ (失效率降低的幅度)都是固定的

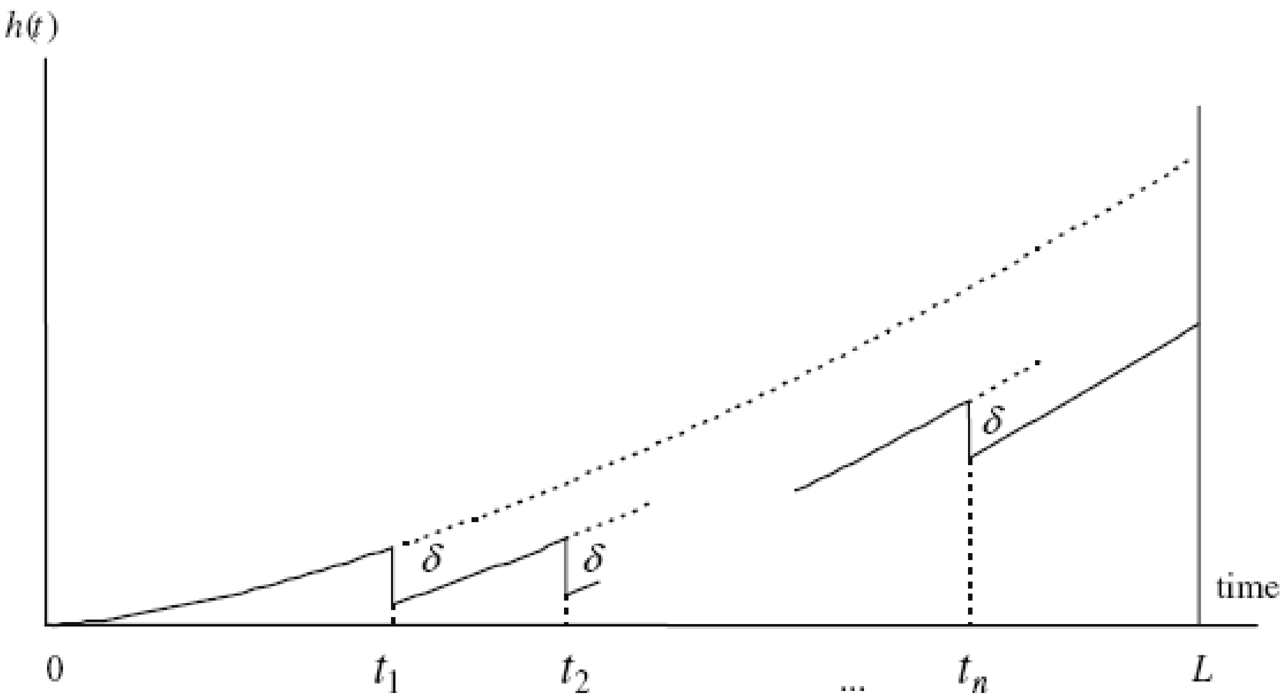

Fig. 1. Preventive maintenance scheme under fixed failure rate reduction.

Mathematical formulation

- 過去學者指出
  - 未進行預防維護動作，設備失效過程為一NHPP
  - 有進行預防維護動作，設備失效過程在$[t_i,t_{i+1}]$區間仍為一NHPP
- 根據上述，在租賃期間有執行維護動作下，其預期的失效數為

$$\Lambda \equiv \Lambda(n,\delta,\underline{\mathbf{t}}) = \sum_{i=0}^{n}\int_{t_i}^{t_{i+1}}[h(t)-i\delta]\mathrm{d}t = H(L)-\delta\sum_{i=1}^{n}(L-t_i), \qquad (1)$$

Mathematical formulation

小維修

固定處罰

維修超過約定時間處罰

PM

$$C(n,\delta,\underline{\mathbf{t}}) = [C_m + C_n + C_\tau G(\overline{\tau})]\Lambda + nC_{pm}(\delta)$$
$$= KH(L) + nC_{pm}(\delta) - K\delta\sum_{i=1}^{n}(L - t_i), \qquad (2)$$

where $K = C_m + C_n + C_\tau \int_\tau^\infty \overline{G}(t)\mathrm{d}t$ represents the expected cost for each failure. Without PM actions ($n = 0$), the expected total cost reduces to

$$C_0 \equiv C(0, 0, \underline{\mathbf{t}}; L) = KH(L). \qquad (3)$$

Optimal PM policy

$nC_{pm}(\delta)$ and $K\delta\sum_{i=1}^{n}(L-t_i)$

預防維護成本

預期失效成本

Trade off

Optimal PM policy

$$\text{Minimize } C(n, \delta, \underline{\mathbf{t}}) = C_0 + nC_{pm}(\delta) - K\delta \sum_{i=1}^{n} (L - t_i)$$

$$\text{Subject to } h(t_i) - i\delta \geq 0 \quad \text{for all} \quad i = 1, 2, \ldots, n.$$

theorem1

Theorem 1. *Given any $n > 0$ and $\delta > 0$, if $h(t)$ is a strictly increasing function of t, then $t_i^* = h^{-1}(i\delta)$.*

$$C(n, \delta | \underline{\mathbf{t}}^*) = C_0 + nC_{pm}(\delta) - K\delta\left\{ nL - \sum_{i=1}^{n} [h^{-1}(i\delta)] \right\}.$$

theorem2

Theorem 2. *Given any $n > 0$, the following results hold:*

(i) *If* $b - KL \geqslant 0$, *then* $\delta_n^* = 0$.

(ii) *If* $b - KL < 0$ *and* $2\left(\frac{\partial \sum_{i=1}^{n} h^{-1}(i\delta)}{\partial \delta}\right) + \delta\left(\frac{\partial^2 \sum_{i=1}^{n} h^{-1}(i\delta)}{\partial \delta^2}\right) > 0$, then there exists a unique $\delta_n^* \in \left[0, \frac{h(L)}{n}\right]$ *such that the expected total cost is minimized.*

透過下面步驟求得最佳預防維護策略

Step 1: If $b - KL \geqslant 0$, then $(n^*, \delta^*, \underline{\mathbf{t}}^*) = (0, 0, \mathbf{0})$ and STOP.

Step 2: Set $(n^*, \delta^*, \underline{\mathbf{t}}^*) = (0, 0, \mathbf{0})$, $C(n^*, \delta^*, \underline{\mathbf{t}}^*) = C_0$, $\bar{n} = \left[\frac{L}{\tau}\right]$, and n = 1.

Step 3: Search for $\delta_n^* \in \left(0, \frac{h(L)}{n}\right)$ such that $C(n, \delta_n^* | \underline{\mathbf{t}}^*) = \min_{\delta} C(n, \delta | \underline{\mathbf{t}}^*)$.

Step 4: If $C(n, \delta_n^* | \underline{\mathbf{t}}^*) < C(n^*, \delta^*, \underline{\mathbf{t}}^*)$, then set $C(n^*, \delta^*, \underline{\mathbf{t}}^*) = C(n, \delta_n^* | \underline{\mathbf{t}}^*)$ and $(n^*, \delta^*, \underline{\mathbf{t}}^*) = (n, \delta_n^*, \underline{\mathbf{t}}^*)$.

Step 5: If $n = \bar{n}$, then STOP; otherwise, set $n = n + 1$ and go to Step 3.

Weibull case

$$\text{Minimize } C(n, \delta, \underline{\mathbf{t}}) = K(\alpha L)^{\beta} + n(a + b\delta) - nK\delta L + K\delta \sum_{i=1}^{n} t_i$$

$$\text{Subject to } h(t_i) - i\delta \geq 0 \quad \text{for all} \quad i = 1, 2, \ldots, n.$$

$$\begin{aligned}
&\text{(i) } \mathit{If}\ b - KL \geqslant 0,\ \mathit{then}\ \delta_n^* = 0.\\
&\text{(ii) } \mathit{If}\ b - KL < 0,\ \mathit{then}\ \delta_n^* = \left\{ n\left(L - \tfrac{b}{K}\right)\left(\tfrac{1}{w}\right)\left(\frac{1}{\sum_{i=1}^{n} \frac{1}{i^{\beta-1}}}\right)\left(\tfrac{\beta-1}{\beta}\right) \right\}^{(\beta-1)}.
\end{aligned}$$

Weibull case

- Step 1: If $b - KL \geqslant 0$, then $(n^*, \delta^*, \underline{\mathbf{t}}^*) = (0, 0, \mathbf{0})$ and STOP; otherwise, set $n = 1$.
- Step 2: Set $\delta_n^* = \left\{ n\left(L - \frac{b}{K}\right)\left(\frac{1}{w}\right)\left(\frac{1}{\sum_{i=1}^{n} \frac{1}{i^{\beta-1}}}\right)\left(\frac{\beta-1}{\beta}\right) \right\}^{(\beta-1)}$ and $t_i^* = w(i\delta)^{\frac{1}{\beta-1}}$.
- Step 3: If $n = \bar{n}$, then STOP; otherwise, set $n = n + 1$ and go to Step 2.

Numerical examples for the weibull case

β	μ	L	C_n	C_0	n^*	C^*	δ^*	$\Delta\%$
1.5	0.90275	0.5	0	78.42	0	78.42	0	0
			200	149.13	0	149.13	0	0
		1	0	221.80	0	221.80	0	0
			200	421.80	1	320.27	0.8131	24.1
		3	0	1152.52	2	630.90	0.9123	47.6
			200	2191.75	3	837.56	0.6805	61.8
		5	0	2479.82	3	924.48	0.8760	62.7
			200	4715.89	5	1256.33	0.5769	73.4
2	0.88623	0.5	0	55.45	0	55.45	0	0
			200	105.45	0	105.45	0	0
		1	0	221.80	0	221.80	0	0
			200	421.80	1	357.94	0.8815	15.1
		3	0	1996.22	3	1015.60	1.3873	49.1
			200	3796.22	5	1377.76	0.9605	63.7
		5	0	5545.04	6	1811.06	1.3642	67.3
			200	10545.04	9	2399.17	0.9763	77.2
3	0.89298	0.5	0	27.73	0	27.73	0	0
			200	52.73	0	52.73	0	0
		1	0	221.80	0	221.80	0	0
			200	421.80	1	393.41	1.0360	6.7
		3	0	5988.65	6	2712.31	3.1494	54.7
			200	11388.65	10	3504.32	2.1929	69.2
		5	0	27725.22	17	7099.88	3.7202	74.4
			200	52725.22	24	8761.85	2.8147	83.4

Compared with two other policies

β	C_r	C_n	Policy J			Policy P			Policy Y		
			n^*	C^*	$\Delta\%$	n^*	C^*	$\Delta\%$	n^*	C^*	$\Delta\%$
1.5	0	0	2	615.31*	45.0	-	-	-	2	620.90	44.5
	0	200	4	1042.31*	68.9	-	-	-	4	1065.91	68.2
	300	0	3	907.63*	63.4	-	-	-	3	924.48	62.7
	300	200	5	1223.91	74.0	-	-	-	5	1256.33	73.4
2	0	0	4	1280.00*	48.8	3	1300.00	48.0	4	1280.00*	48.8
	0	200	7	2067.71*	72.4	7	2075.00	72.3	7	2067.71*	72.4
	300	0	6	1811.05*	67.3	6	1820.70	67.2	6	1811.06*	67.3
	300	200	9	2399.16*	77.2	9	2404.50	77.2	9	2399.17*	77.2
3	0	0	10	5437.03*	56.5	9	5750.00	54.0	10	5477.39	56.2
	0	200	20	7712.87*	79.4	21	8034.90	78.6	20	7827.84	79.1
	300	0	16	7009.92*	74.7	17	7312.50	73.6	17	7099.88	74.4
	300	200	24	8610.66*	83.7	25	8969.90	83.0	24	8761.85	83.4

Conclusion

- 本研究採失效率降低方法，為租賃設備提供一最佳預防維護策略。
- 從這個韋伯例子中
  - 得到一closed-form solution
  - 最佳維護策略中，採固定維護程度較優於採定期維護間隔好

未來延伸方向

如非線性維修費用，時間依賴性處罰成本，或各種處罰辦法，擴大問題的未來研究這個問題區。