Control Charts for Monitoring Fault Signatures: Cuscore versus GLR

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Key Word: generalized likelihood ratio; cumulative score; CUSUM; signal detection; statistical process control

Abstract(1/2)

- A process mean change is not persistenly constant but time varying.
- Two control charts of this kind: generalized likelihood ratio(GLR) and cumulative score(Cuscore).

Abstract(2/2)

- Investigation purposes: a sine wave representing a bounded signal and a linear trend representing an unbounded signal.
- Cases analysis: a known fault signature and parameter and a known fault signature but unknown parameter.

Introduction(1/2)

- Fault signatures in the form of linear, exponential,or sinusoidal patterns are commom in manufacturing processes.
- The key role of restarts used in Cuscore charts by evaluating their performance for fault signatures starting at time zero and at unknown time

Introduction(2/2)

- Fisher RA(1925): The cuscore control statistic is based on the concept of Fisher's efficient scores.
- Luceno(1999): Used a CUSUM-like restart procedure and provided algorithms to compute average run lengths(ARLs) and corresponding run-length probability distributions for Cuscore charts to control a process mean.

Process model

$$y_t = \mu + a_t, \quad t = 0, 1, 2, \ldots \qquad (1)$$

$$y_t = \mu + f(t, \theta, \tau) + a_t, \quad t = 0, 1, 2, \ldots \qquad (2)$$

Notation:

Y : the quality characteristic of interest

μ : the mean of Y

a_t : the normally distributed white - noise sequence
with mean zero and standard deviation σ

t : the sequence order or time

τ : unknown time

θ : the amplitude of fault signature

Cuscore charts(1/2)

$$S_t = \max[S_{t-1} + (y_t - \mu - k_t) f(t, \theta, \tau); 0], \quad t = 1, 2, \ldots \qquad (3)$$

Notation:

k_t : handicap

d : a step shift with magnitude

- Cumulative sum charts are a specific case of Cuscore charts.
- Handicap k_t is usually chosen proportional to the signal value.

Cuscore charts(2/2)

- In statistical process monitoring a signal often dose not occur until some later time $\tau > 0$.
- This reinitalization prevents the Cuscore statistic from decreasing when there is no hidden signal in the date.

GLR charts(1/2)

- Fault detection method:

$$l_t(\theta, \tau) = \sum_{i=j}^{t} \ln \frac{p_{\mu+f(i,\theta,j)}(y_i)}{p_{\mu}(y_i)} \qquad (4)$$

Notation:

p(.): the parameterized joint probability of the observation

- Fault starting point:

$$g_t(\theta) = \max_{1 \leqslant j \leqslant t} \sum_{i=j}^{t} \ln \frac{p_{\mu+f(i,\theta,j)}(y_i)}{p_{\mu}(y_i)} \qquad (5)$$

GLR charts(2/2)

- GLR statistic:

$$g_t = \max_{1 \leqslant j \leqslant t} \left[\frac{1}{\sigma^2} \sum_{i=j}^{t} f(i, \theta, j)(y_i - \mu) - \frac{1}{2\sigma^2} \sum_{i=j}^{t} f(i, \theta, j)^2 \right] \quad (6)$$

- GLR with a moving window of size w:

$$g_t = \max_{t-w \leqslant j \leqslant t} \left[\frac{1}{\sigma^2} \sum_{i=j}^{t} f(i, \theta, j)(y_i - \mu) - \frac{1}{2\sigma^2} \sum_{i=j}^{t} f(i, \theta, j)^2 \right] \quad (7)$$

Notation:
w: the most recent time periods

An illustrative example

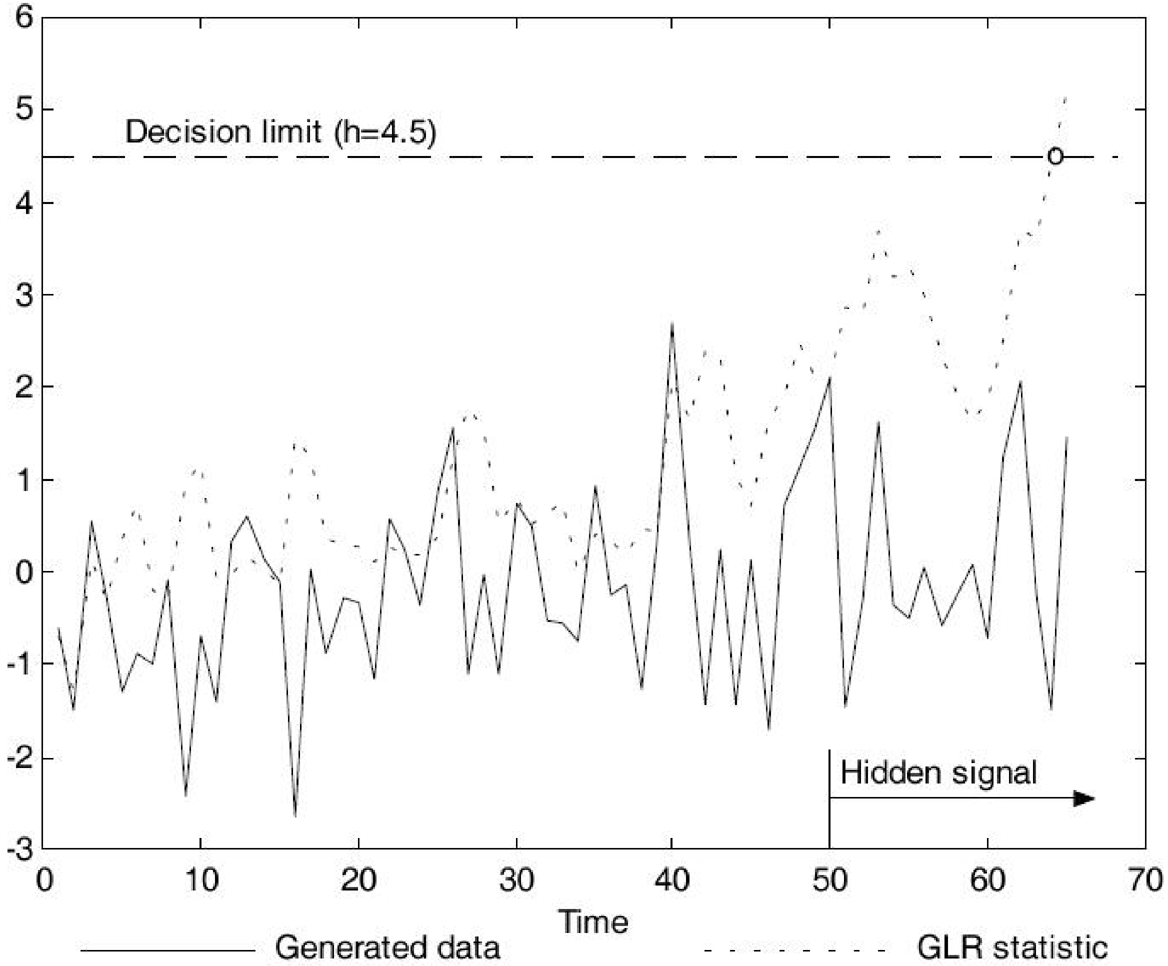

Figure 1. GLR control chart for a $\sin[(t-0.5)\pi/2]$ signal

Unknown magntude of a fault signature(1/2)

- the signal form is known but the true value of the signal parameter θ is unknown.
  Such that: $f(t, \theta, \tau) = \theta(t - \tau)$ (9)
- Parameter estimator:

$$\hat{\theta}_{t(j)} = \frac{\sum_{i=j}^{t} y(i) h(i, j)}{\sum_{i=j}^{t} h(i, j)^2}, \quad t - w \leqslant j \leqslant t \tag{10}$$

Notation：

$h(i, j)$: the value of the signal at time i assuming the signal started at time $\tau = j$

Unknown magntude of a fault signature(2/2)

- Equation (3)&(5) can be written as:

$$S_t = \max\{S_{t-1} + (y_t - k_t) f(t, \hat{\theta}_{t(j)}, \tau); 0\}, \quad t = 1, 2, \ldots \tag{11}$$

$$g_t = \max_{t-w \leqslant j \leqslant t} \left[\frac{1}{\sigma^2} \sum_{i=j}^{t} f(i, \hat{\theta}_{t(j)}, j)(y_i - \mu) - \frac{1}{2\sigma^2} \sum_{i=j}^{t} f(i, \hat{\theta}_{t(j)}, j)^2 \right]$$

- A lower bound(or upper bound) can be used to reduce the imprecision.

$$\max(\hat{\theta}_{t(j)}, \theta_{\mathrm{L}}) \quad \text{or} \quad \min(\hat{\theta}_{t(j)}, \theta_{\mathrm{U}}) \tag{13}$$

Performance comparisons(1/3)

- In-control ARL(ARL_0)/Out-of-control ARL(ARL_1): the average number of time units to falsely/correctly alarm the presence of a specific signal in the process date.
- The GLR have a slightly better performance over the Cuscore charts with reinitialization.

Table I. ARLs for known parameter case for linear trend with slope 0.1 (standard errors are given in brackets)

		GLR		Cuscore (reinitialized)	
	Signal starts at	$h = 2.5$	$h = 3.0$	$h = 1.9$	$h = 2.35$
ARL(0)		170.5 (2.32)	285.46 (3.79)	170.24 (2.41)	286.5 (4.05)
ARL(1)	$\tau = 0$	11.20 (0.04)	11.85 (0.04)	11.42 (0.04)	12.13 (0.04)
ARL(1)	$\tau = 50$	10.38 (0.05)	11.23 (0.05)	10.91 (0.05)	11.70 (0.05)

Performance comparisons(2/3)

- The Cuscore charts have a very good performance for the signal starting at $\tau = 0$ .When the signalis started at $\tau = 50$,Cuscore performance is poor without reinitialization.

Table II. ARLs for known parameter case for $\sin[(t - 0.5)\pi/2]$ (standard errors are given in brackets)

		GLR			Cuscore		
	Signal starts at	$h = 4.0$	$h = 4.5$	$h = 5$	$h = 3.1$	$h = 3.65$	$h = 4.1$
ARL(0)		188.68 (2.51)	323.83 (4.52)	523.37 (7.32)	184.6 (2.50)	332.43 (4.57)	528.48 (7.44)
ARL(1)	$\tau = 0$	16.08 (0.13)	18.56 (0.15)	20.63 (0.17)	12.06 (0.17)	13.88 (0.20)	15.91 (0.23)
ARL(1)	$\tau = 50$	15.00 (0.14)	17.06 (0.24)	19.22 (0.17)	137.55 (1.95)	248.33 (3.55)	407.68 (5.76)

Cuscore (reinitialized)		
$h = 3.1$	$h = 3.65$	$h = 4.1$
183.58 (2.60)	330.82 (4.37)	524.25 (7.40)
18.71 (0.26)	21.8 (0.22)	23.14 (0.22)
19.83 (0.21)	22.93 (0.24)	24.78 (0.24)

Performance comparisons(3/3)

Table III. ARLs for unknown slope case for linear trend θt with $\theta_L = 0.05$ (standard errors are given in brackets)

	Signal starts at	GLR $h = 4$	GLR $h = 4.5$	Cuscore (reinitialized) $h = 6.7$	Cuscore (reinitialized) $h = 7.5$
ARL(0)		189.42 (2.67)	306.75 (4.30)	183.16 (2.33)	311.36 (3.97)
ARL(1)	$\tau = 0$	11.46 (0.05)	12.21 (0.05)	13.32 (0.04)	14.17 (0.04)
ARL(1)	$\tau = 50$	11.24 (0.05)	11.98 (0.05)	13.35 (0.05)	14.21 (0.05)

Table IV. ARLs for unknown amplitude case for $\theta \sin[(t - 0.5)\pi/2]$ with $\theta_L = 0.5$ (standard errors are given in brackets)

	Signal starts at	GLR $h = 5$	GLR $h = 5.5$	Cuscore (reinitialized) $h = 4.1$	Cuscore (reinitialized) $h = 4.7$
ARL(0)		216.72 (2.99)	357.36 (4.99)	213.05 (3.04)	350.87 (5.00)
ARL(1)	$\tau = 0$	17.61 (0.16)	20.09 (0.18)	19.12 (0.31)	22.23 (0.36)
ARL(1)	$\tau = 50$	17.58 (0.16)	19.08 (0.18)	29.60 (0.41)	34.40 (0.47)

- The GLR outperforms Cuscore in all of the cases.

Conclusions

- The GLR charts are based upon a simple maximum likelihood derivation that is easily applied to other cases.
- The GLR avoid the reintialization issue that affects the performance of Cuscore charts.
- The GLR provide a method to incorporate unknown parameters within a common framework.