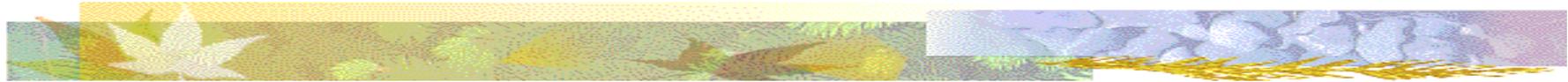




A New Strategy for Phase I Analysis in SPC



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Keywords: Bonferroni's adjustment; individual false-alarm rate; multiple tests
overall false-alarm rate; signal probability

Introduction (1/3)

- 統計製程管制SPC上，管制圖常被區分為階段一和階段二。
 - 階段一管制圖從一組所謂歷史資料去追溯，以離線(off-line)方式決定製程是否已在管制內；階段二管制圖主要以線上(online)方式對連續製程作監控。階段一製程資料隨著將製程移至一統計管制狀態目的而被收集與分析，對於稍後階段二線上製程監控完成可信賴管制界限。
- 對於線上製程監控去建立有效管制圖，選擇合適製程(產品)品質相關統計量和根據管制內分配去建立管制界限。階段一管制圖通常從資料組刪除管制外資料，以致假定合理管制內製程資料能獲得去建立監控統計量分配。

Introduction (2/3)

- 對於階段一傳統分析作法，discard-all程序傾於誤刪過多管制內資料點，為了解決此問題，對於收集管制內資料提出有效率簡化拋棄疊代程序來取代—一次篩選一個 (one-at-a-time; OAAT)。
- 所提出OAAT程序，與傳統程序相比似乎有較多疊代。建議在整個OAAT程序結束，對於所有管制外所被偵測點去執行研究。注意，最後管制外點數至多等於傳統discard-all程序，因此由減少調查頻率和可能製程修正，達到有效成本節省。
- 訊號機率(singal probability)定義為族(family-wise)訊號率即在資料組中至少一樣本點發出管制外訊號。

Introduction (3/3)

- 早期，根據控制各樣本個別誤警率來設計管制圖，近來許多階段一研究根據控制整體誤警率。在固定個別誤警率下，當樣本數 m 增加，整體誤警率亦變大，這也是為何作者選擇由Bonferroni-type程序來修正個別誤警率，以控制所希望整體誤警率水準。

The conventional method (1/5)

Estimating process parameter

$$\hat{\mu}_0 = \bar{\bar{X}} = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n X_{ij} = \frac{1}{m} \sum_{i=1}^m \bar{X}_i$$

$$\hat{\sigma}_0 = \frac{\bar{R}}{d_2} = \frac{\frac{1}{m} \sum_{i=1}^m R_i}{d_2}, R_i = \max\{X_{i1}, \dots, X_{in}\}; d_2 = \frac{E(R)}{\sigma_0}$$

$$= \frac{\bar{S}}{c_4} = \frac{\frac{1}{m} \sum_{i=1}^m S_i}{c_4}, S_i = \left(\frac{\sum_{j=1}^n (X_{ij} - \bar{X}_i)^2}{(n-1)} \right)^{1/2}; c_4 = \frac{\sqrt{2}\Gamma\left(\frac{n}{2}\right)}{\sqrt{n-1}\Gamma\left(\frac{(n-1)}{2}\right)}$$

$$= \frac{\bar{V}^{1/2}}{c_{4,m}} = \frac{\left(\frac{1}{m} \sum_{i=1}^m S_i^2 \right)^{1/2}}{c_{4,m}}, c_{4,m} = \frac{\sqrt{2}\Gamma\left(\frac{(m(n-1)+1)}{2}\right)}{\sqrt{m(n-1)}\Gamma\left(\frac{m(n-1)}{2}\right)}$$

The conventional method (2/5)

The individual and overall false-alarm rates

$$\bar{X}_i - \bar{\bar{X}} \sim N\left(0, \frac{(m-1)\sigma_0^2}{(mn)}\right)$$

$$m(n-1)\bar{V} / \sigma_0^2 \sim \chi_{m(n-1)}^2$$

$$\frac{\sqrt{mn}(\bar{X}_i - \bar{\bar{X}})}{\sqrt{(m-1)}\sqrt{\bar{V}}} = \frac{\sqrt{mn}(\bar{X}_i - \bar{\bar{X}})}{\sqrt{(m-1)\sigma_0^2}} \Big/ \frac{\sqrt{\bar{V}}}{\sqrt{\sigma_0^2}} \sim t_{m(n-1)}$$

$$\text{當 } \frac{\sqrt{mn}(UCL - \bar{\bar{X}})}{\sqrt{(m-1)}\sqrt{\bar{V}}} = \frac{\sqrt{mn}\left(\frac{k\hat{\sigma}_0}{\sqrt{n}}\right)}{\sqrt{(m-1)}\sqrt{\bar{V}}} = \frac{k\sqrt{m}}{C_{4,m}\sqrt{m-1}}$$

$$\text{則 } \alpha^* = 2\left(1 - F_{t_{m(n-1)}}\left(\frac{k\sqrt{m}}{C_{4,m}\sqrt{m-1}}\right)\right)$$

The conventional method (3/5)

Table I. The individual false-alarm rate, α^* , and the overall false-alarm rate, α , for various values of m and n when $k=3$, assuming m tests are independent

m	n	$c_{4,m}$	α^*	α
30	5	0.9979	0.0028	0.0793
30	10	0.9991	0.0025	0.0719
30	15	0.9994	0.0024	0.0698
50	5	0.9988	0.0027	0.1278
50	10	0.9994	0.0026	0.1207
50	15	0.9996	0.0025	0.1187
100	5	0.9994	0.0027	0.2381
100	10	0.9997	0.0026	0.2318
100	15	0.9998	0.0026	0.2300

- 1.因設定 $k = 3$ 之故，所以個別誤警率 α^* 並無多大差異；但整體誤警率 $\alpha = 1 - (1 - \alpha^*)^m$ ，隨著 m 增加或 n 減少時則增加。
- 2.Jones&Champ(2002)指出，階段一管制圖其獨立假設並非成立，但幸好假設違反無顯著影響在誤警率上。

The conventional method (4/5)

Excessive false-alarm rates

Table II. The individual false-alarm rate, α^{**} , for the case of $k=3$ and $n=5$. p is the proportion of the out-of-control process with shift size $\delta = (\mu_1 - \mu_0) / \sigma_0$

δ	$p=0.1$	$p=0.2$	$p=0.3$	$p=0.4$	$p=0.5$
0.4	0.002807	0.003132	0.003692	0.004511	0.005626
0.8	0.003132	0.004511	0.007085	0.011274	0.017670
1.2	0.003692	0.007085	0.014152	0.027032	0.048630
1.6	0.004511	0.011274	0.027032	0.058338	0.112921
2.0	0.005626	0.017670	0.048630	0.112921	0.222454
2.4	0.007085	0.027032	0.082262	0.196726	0.375729
2.8	0.008945	0.040260	0.130995	0.310087	0.551913
3.2	0.011274	0.058338	0.196726	0.445186	0.718270
3.6	0.014152	0.082262	0.279258	0.587040	0.847300
4.0	0.017670	0.112921	0.375729	0.718270	0.929508

$$1.\alpha^{**} = P(\bar{X} > UCL \text{ or } \bar{X} < LCL | \mu = \mu_0) = 1 - \Phi(\sqrt{n}p\delta + k) + \Phi(\sqrt{n}p\delta - k)$$

The conventional method (5/5)

Table III. The detecting power of an individual test, $1 - \beta^{**}$, for the case of $k=3$ and $n=5$. p is the proportion of the out-of-control process with shift size $\delta = (\mu_1 - \mu_0) / \sigma_0$

δ	$p=0.1$	$p=0.2$	$p=0.3$	$p=0.4$	$p=0.5$
0.4	0.014152	0.011274	0.008945	0.007085	0.005626
0.8	0.082262	0.058338	0.040260	0.027032	0.017670
1.2	0.279258	0.196726	0.130995	0.082262	0.048630
1.6	0.587040	0.445186	0.310087	0.196726	0.112921
2.0	0.847300	0.718270	0.551913	0.375729	0.222454
2.4	0.966368	0.902038	0.775353	0.587040	0.375729
2.8	0.995792	0.977720	0.916621	0.775353	0.551913
3.2	0.999709	0.996778	0.977720	0.902038	0.718270
3.6	0.999989	0.999709	0.995792	0.966368	0.847300
4.0	1.000000	0.999984	0.999445	0.991023	0.929508

$$2. 1 - \beta^{**} = P(\bar{X} > UCL_0 \text{ or } \bar{X} < LCL_0 | \mu = \mu_1) = 1 - \Phi(-\sqrt{n}(1-p)\delta + k) + \Phi(-\sqrt{n}(1-p)\delta - k)$$

3. 當固定偏移大小 δ 時，若 p 從 0 增加，則誤警率從 0.0027 增加，此說明為何當資料受到污染其將所超過預期錯誤警報次數；而檢定力則減少，顯示 discard-all 程序快速損失檢定力。

New strategy and the OAAT method for Phase I analysis (1/5)

Criteria for performance evaluation

- 關於檢定階段一 m 個樣本數管制內與否問題，分別定義 m_0 和 m_1 為管制內和管制外之樣本數。假設有 R 個樣本數被拒絕，其中 R_0 為管制內樣本被錯誤拒絕數目和 R_1 為管制外樣本被正確拒絕數目。
- 將製程移至一統計管制狀態和收集管制內資料之目的為，不僅要知道製程是否於管制內，並當答案為“否”時，更需知道哪個樣本為管制外，但訊號機率僅能提供前述問題之量測。

New strategy and the OAAT method for Phase I analysis (2/5)

- 當歷史資料組包含管制內和管制外之資料混合型式時 ($m > m_1 > 0$)，訊號機率 P 不為整體誤警率 (除非 $m_1 = 0$)，也亦非檢定力 (除非 $m_1 = m$)，且管制外訊號能被真實或錯誤警報其所觸發。換言之，高訊號機率能被高檢定力或高誤警率其一或兩者同時造成而無法辨識。

A new strategy on when to inspect

- 前述所提，各疊代在管制圖架構下，可能存在些許管制外樣本尚未被偵測到，尤其當試驗管制界限由一組包含管制外樣本所計算而得，將造成 stop-and-inspect 動作不確定性過高。因此建議在經歷所有疊代程序後，最後對於所有管制外點，執行非隨機原因檢查。

New strategy and the OAAT method for Phase I analysis (3/5)

An illustrative simulation study of discard-all practice

Table IV. \hat{P} , \bar{R}_0 , and \bar{R}_1 of the discard-all procedure based on 1000000 replications for various values of m_1 and δ when $m=30$, $n=5$, and $k=3$

m_1	δ	\hat{P}	\bar{R}_0/m_0	\bar{R}_1/m_1
0	0	0.0780	0.0828/30	0/0
3 (10%)	0.4	0.1104	0.0769/27	0.0426/3
	0.8	0.2853	0.0867/27	0.2478/3
	1.2	0.6530	0.1011/27	0.8394/3
	1.6	0.9336	0.1241/27	1.7609/3
	2	0.9965	0.1539/27	2.5399/3
	2.4	1	0.1940/27	2.8986/3
	2.8	1	0.2445/27	2.9872/3
	3.2	1	0.3085/27	2.9991/3
	3.6	1	0.3864/27	3.0000/3
	4	1	0.4820/27	3.0000/3
6 (20%)	0.4	0.1322	0.0767/24	0.0684/6
	0.8	0.3651	0.1101/24	0.3512/6
	1.2	0.7590	0.1726/24	1.1836/6
	1.6	0.9729	0.2742/24	2.6721/6
	2	0.9995	0.4276/24	4.3063/6
	2.4	1	0.6549/24	5.4090/6
	2.8	1	0.9736/24	5.8643/6
	3.2	1	1.4109/24	5.9802/6
	3.6	1	1.9864/24	5.9981/6
	4	1	2.7252/24	5.9999/6

1. 對於 $m_1 = 0$ ，則 \hat{P} 為整體誤警率，大約 0.078；另一方面所估計誤警率 \bar{R}_0 為 0.0828

大於 \hat{P} ，原因為當 $R_0 \geq 1$ ，無論多少錯誤警報存在資料組中，對於估計 P 只能計數到 1。

New strategy and the OAAT method for Phase I analysis (4/5)

Table IV. \hat{P} , \bar{R}_0 , and \bar{R}_1 of the discard-all procedure based on 1000000 replications for various values of m_1 and δ when $m=30$, $n=5$, and $k=3$

m_1	δ	\hat{P}	\bar{R}_0/m_0	\bar{R}_1/m_1
9 (30%)	0.4	0.1442	0.0784/21	0.0815/9
	0.8	0.3926	0.1514/21	0.3652/9
	1.2	0.7664	0.3005/21	1.1841/9
	1.6	0.9715	0.5729/21	2.7942/9
	2	0.9994	1.0277/21	4.9658/9
	2.4	1	1.7382/21	6.9747/9
	2.8	1	2.7604/21	8.2450/9
	3.2	1	4.1422/21	8.7973/9
	3.6	1	5.8693/21	8.9612/9
12 (40%)	0.4	0.1521	0.0829/18	0.0870/12
	0.8	0.3979	0.2054/18	0.3276/12
	1.2	0.7475	0.4906/18	0.9919/12
	1.6	0.9582	1.0576/18	2.3681/12
	2	0.9983	2.0402/18	4.5105/12
	2.4	1	3.5516/18	7.0427/12
	2.8	1	5.5863/18	9.2968/12
	3.2	1	8.0168/18	10.8186/12
	3.6	1	10.5676/18	11.5927/12
4	1	12.9224/18	11.8906/12	

2. \bar{R}_1 隨著 δ 值增加而增加；且 \bar{R}_0 亦增加但造成成本不穩定。

3. 當管制外樣本數比例增加，則誤警率 (\bar{R}_0/m_0) 增加。

4. \bar{R}_0 和 \bar{R}_1 之變動方向如同 \hat{P} 值；當 δ 值為 2.4 時，則 \hat{P} 值到達 1，然而 \bar{R}_0 和 \bar{R}_1 仍持續成長。

New strategy and the OAAT method for Phase I analysis (5/5)

The OAAT method

1. 由所被收集之全部資料去建立試驗管制界限。
2. 如果由所定義管制界限判定為無管制外樣本數時，則停止疊代並跳至步驟4，否則刪除最極端樣本值。
3. 由剩餘樣本數去建立試驗管制界限，並跳回步驟2。
4. 如果已無管制外樣本數可刪除，則製程可宣示為管制內；另外收集上述疊代所刪除樣本數去檢查製程非隨機性原因。

The performance of the OAAT procedure (1/5)

Controlling the individual false-alarm rate

Table V. Average numbers of false alarms, \bar{R}_0 , of the discard-all procedure and the OAAT procedure when all the samples are *in control* for various combinations of m and n . Here $k=3$

m	n	Discard-all	OAAT
30	5	0.0834	0.0818
30	10	0.075	0.0738
30	15	0.0726	0.0716
50	5	0.1384	0.1362
50	10	0.1291	0.1273
50	15	0.1271	0.1254
100	5	0.2749	0.2711
100	10	0.2651	0.2617
100	15	0.2639	0.2606

- 1.此研究各方案下， \bar{R}_0 在OAAT程序下均少於discard-all程序，說明了OAAT程序產生較少錯誤警報，但其減少幅度不大，小於2.1%。

The performance of the OAAT procedure (2/5)

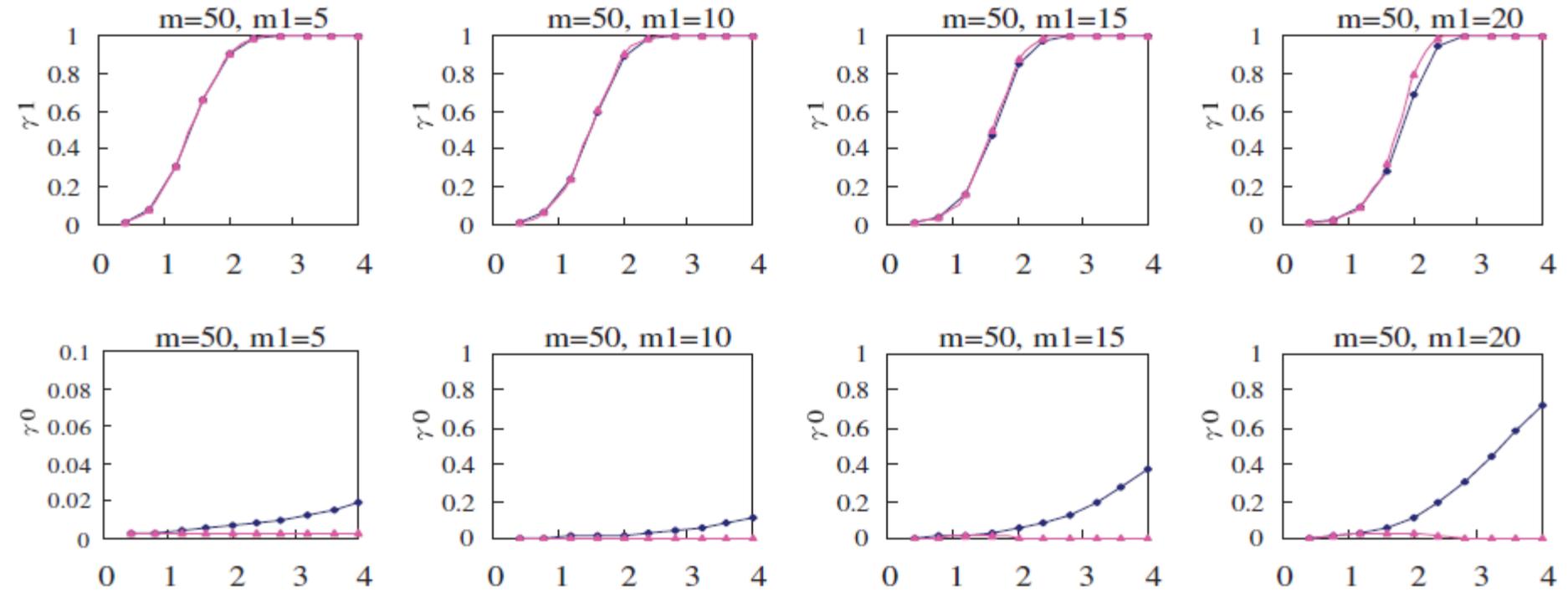


Figure 1. \bar{R}_1/m_1 (γ_1) and \bar{R}_0/m_0 (γ_0) for various combinations of m and m_1 ($n=5$) when $k=3$. The x-axis is the shift size δ . The solid line with diamonds corresponds to the discard-all procedure and the dashed line with triangles corresponds to the OAAT procedure. This figure is available in colour online at www.interscience.wiley.com/journal/qre

1. OAAT程序在檢定力(\bar{R}_1/m_1)方面略優於discard-all程序。
2. OAAT程序在誤警率(\bar{R}_0/m_0)方面一致小於discard-all程序。且OAAT程序 \bar{R}_0/m_0 曲線水平貼近x軸，顯示新OAAT程序稀少錯誤警報產生。
3. 從誤警率而言，OAAT程序改善能力高於discard-all程序，尤其當 m, n 或 p 較大時。

The performance of the OAAAT procedure (3/5)

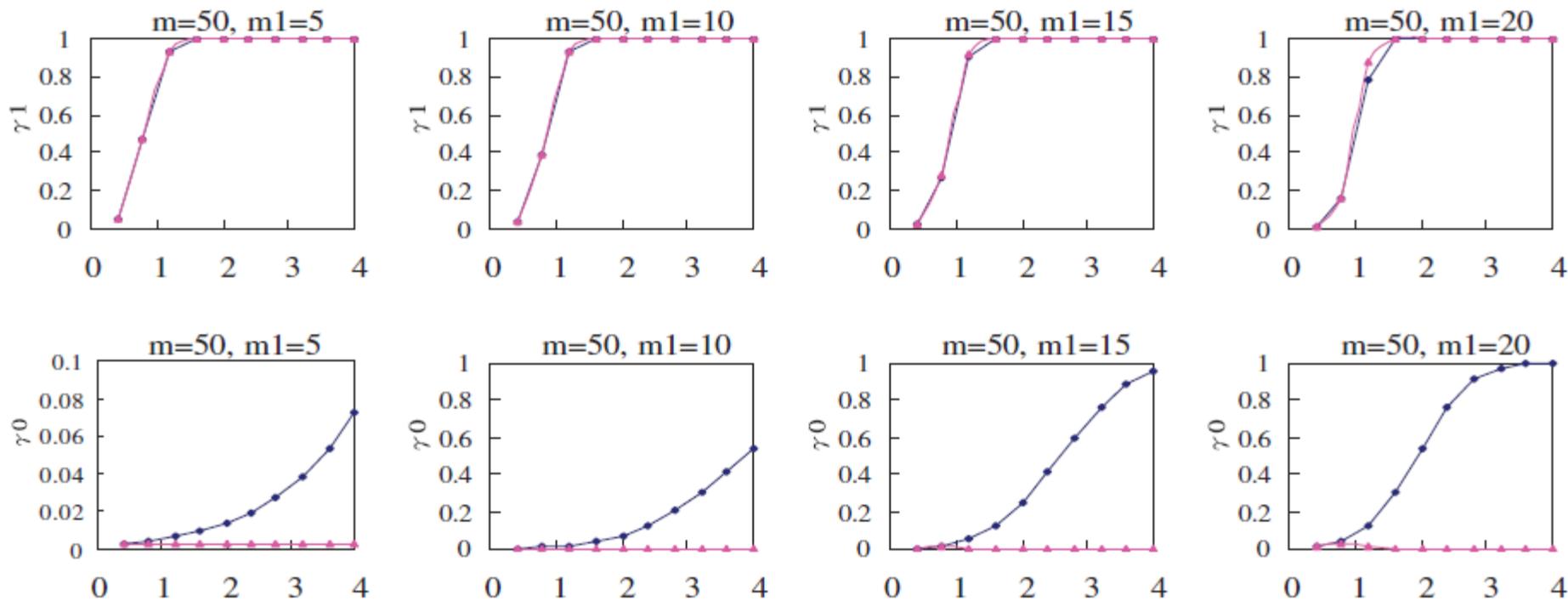


Figure 2. \bar{R}_1/m_1 (γ_1) and \bar{R}_0/m_0 (γ_0) for various combinations of m and m_1 ($n=15$). The x-axis is the shift size δ . The solid line with diamonds corresponds to the discard-all procedure and the dashed line with triangles corresponds to the OAAAT procedure. This figure is available in colour online at www.interscience.wiley.com/journal/qre

- 當 n 增加時，則兩程序之檢定力 (\bar{R}_1/m_1) 增加；對於誤警率，*discard - all* 程序則增加，而多數 *OAAAT* 程序則減少。
- 當 n 和 p 固定時，誤警率 (\bar{R}_0/m_0) 與 m 值變動密切關係。因由設定 $k = 3$ 來控制個別誤警率，即從表一可看出，對於 m 和 n 值變化，理論誤警率並無多大差異。

The performance of the OAAAT procedure (4/5)

Controlling the overall false-alarm rate

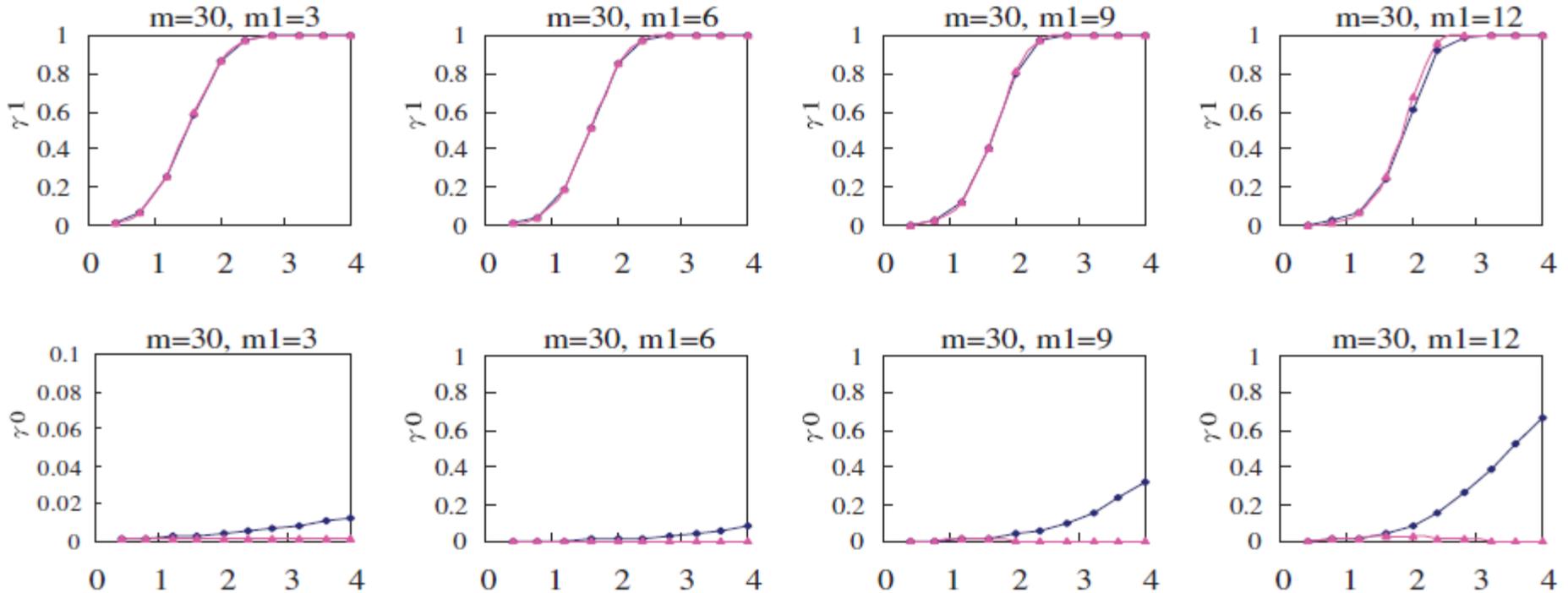


Figure 3. \bar{R}_1/m_1 (γ_1) and \bar{R}_0/m_0 (γ_0) for various combinations of m and m_1 ($n=5$) when the overall false-alarm-rate α is controlled at 0.05 by Bonferroni's adjustment. The x-axis is the shift size δ . The solid line with diamonds corresponds to the discard-all procedure and the dashed line with triangles corresponds to the OAAAT procedure. This figure is available in colour online at www.interscience.wiley.com/journal/qre

1. 利用 *Bonferroni* 方法去控制整體誤警率，令 $k = \sqrt{\frac{(m-1)}{m}} C_{4, m, m(n-1), \alpha/2m}$

k 值隨著 m 增加或 n 減少而增加。

The performance of the OAAT procedure (5/5)

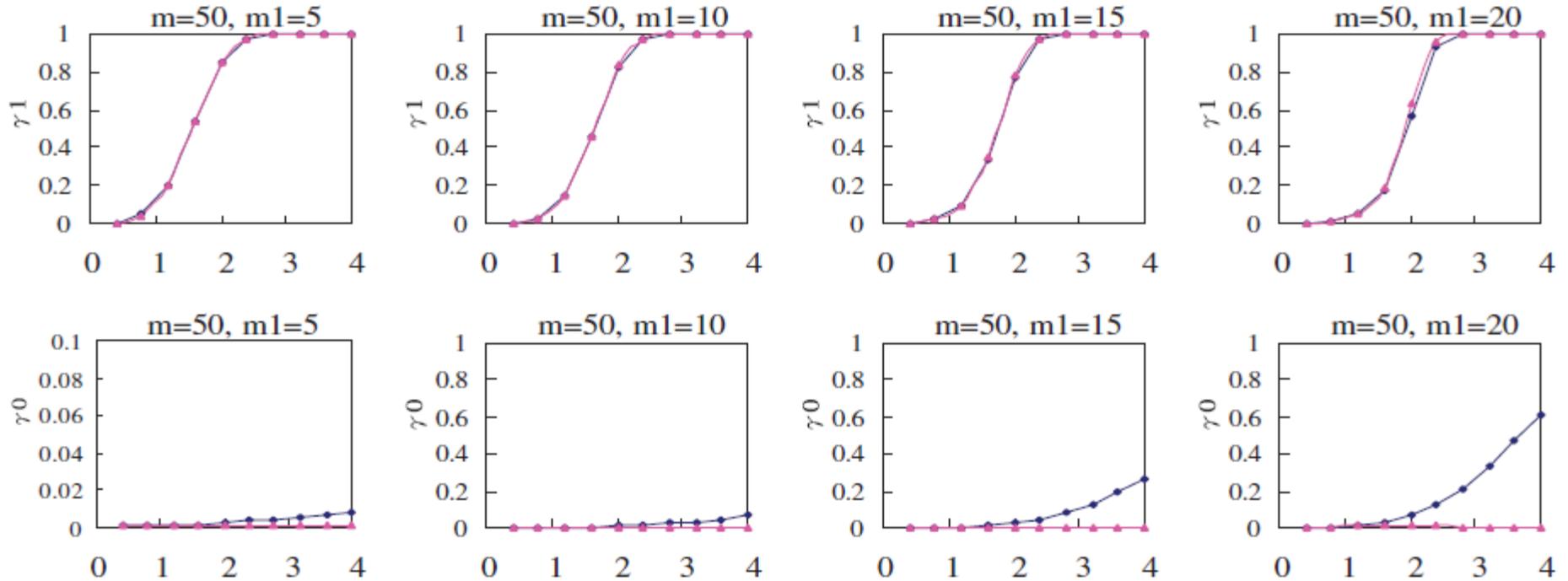


Figure 3. \bar{R}_1/m_1 (γ_1) and \bar{R}_0/m_0 (γ_0) for various combinations of m and m_1 ($n=5$) when the overall false-alarm-rate α is controlled at 0.05 by Bonferroni's adjustment. The x-axis is the shift size δ . The solid line with diamonds corresponds to the discard-all procedure and the dashed line with triangles corresponds to the OAAT procedure. This figure is available in colour online at www.interscience.wiley.com/journal/qre

- 當 m 增加時，則兩程序之檢定力 (\bar{R}_1/m_1) 減少。多重管制意謂著對於 m 個樣本數控制其固定整體誤警率 (α)，當 m 增加，則個別誤警率 (α^*) 減少，造成管制外區域變小。
- 兩程序之檢定力 (\bar{R}_1/m_1) 約略相同，若對於誤警率 (\bar{R}_0/m_0) 而言，OAAT 程序執行上較 discard-all 程序良好，但隨著 m 增加其優勢漸減。

Summary and concluding remarks

- 誤差管制運用兩種方法：個別誤警率和整體誤警率。傳統作法上，在所有疊代過程中，分別對被檢測樣本之個別誤警率保持固定；但當使用Bonferroni's修正去控制整體誤警率，則當更多管制外樣本數被刪除其將變大。
- 不同管制計劃之標準績效評估：文獻所提及訊號機率($P(R \geq 1)$)和本篇paper建議同時使用真實和錯誤警報次數($E(R_0)$ 和 $E(R_1)$)。訊號機率思考製程為”整個”，僅能評價不管製程是否管制內之判斷能力。當資料組具有混和管制內和管制外之樣本數時，可分別同時由誤警率和檢定力去評價。