Design of EWMA and CUSUM control charts subject to random shift sizes and quality impacts.

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Introduction (1/2)

- 對於特定製程偏移下，給予以ARL為根據之管制圖作最佳化，然而製程通常受限於各種誤差大小，相對限制了它應用性。
  - Sparks(2002) 和Capizzi & Masarotto(2003)藉由預測未來偏移大小於管制圖設計之動態修正變動問題。
- 對於製程連續性偏移若跟隨清晰估計點圖(pattern)，其上述設計邏輯為可行，但大多數製程偏移發生為任意且伴隨著隨機大小。

Should a different control chart design be proposed?

Introduction (2/2)

- 由於不同偏移大小將導致不同品質損失(quality losses)，設計邏輯應代入在品質上去思考偏移影響。
  - Loenzen & Vance(1986) et al.在製程偏移預定隨機模式下，管制圖經濟性設計已思考各種品質成本。
- 經濟性設計由於限制了實際使用，設計方法總備受爭議；另一要點在於抽樣成本與品質成本間的權衡。
  - Keat(1997)克服了估計動態成本和建構隨機管制外程序之困難處。

Quality impact induced by control charts (1/3)

$$\mathrm{QL}(X_i) = w(X_i - T)^2$$

由Taguchi所提出二次品質損失函數(quadratic quality loss function)

$$\mathrm{QL}_s = \mathrm{QL}(X_i \mid \mu_X = T + s, \sigma_X)$$

隨機偏移大小s，假設服從具有平均數μ_s和標準差σ_s之常態機率分配函數g(・)且在不失一般性下，令T=0。

$$E(\mathrm{QL}_s) = \int_{-\infty}^{\infty} \mathrm{QL}(x) f(x \mid \mu_X = T + s, \sigma_X)\,\mathrm{d}x$$

$$= E[w(x_i + T)^2 \mid \mu_X = T + s, \sigma_X] = w\left(s^2 + \sigma_X^2\right)$$

Quality impact induced by control charts (2/3)

$$\mathrm{QL}_{1|s} = \sum_{i=1}^{\mathrm{RL}_{1|s}} \mathrm{QL}(X_i \mid \mu_x = T + s, \sigma_x)$$

進一步可以計算在管制外期間($\mathrm{RL}_{1|s}$)所引發品質損失去取代ARL_1。因QL_s為一隨機變數，故$\mathrm{QL}_{1|s}$亦為一隨機總和。

$$\begin{aligned} E(\mathrm{QL}_{1|s}) &= E(\mathrm{RL}_{1|s})E[\mathrm{QL}(X_i \mid \mu_x = T + s, \sigma_x)] \\ &= \mathrm{ARL}_{1|s} \times E(\mathrm{QL}_s). \end{aligned}$$

根據Wald's定理， $\mathrm{RL}_{1|s}$為隨機和之停止點(stopping time)

$$\begin{aligned} E(\mathrm{QL}_1) &= \int_{-\infty}^{\infty} E(\mathrm{QL}_{1|s}) g(s) \mathrm{d}s \\ &= \int_{-\infty}^{\infty} \mathrm{ARL}_{1|s} E(\mathrm{QL}_s) g(s) \mathrm{d}s. \end{aligned}$$

計算分為兩種：Monte Carlo simulation 和numerical approximation(包含Markov chain 與integral equation)

Quality impact induced by control charts (3/3)

$$\begin{aligned}\mathrm{ARL}^{+}(u) &= 1+\mathrm{ARL}^{+}(0)F(K-u)\\ &\quad+\int_{0}^{h\sigma_x}\mathrm{ARL}^{+}(x)f(x+K-u)\mathrm{d}x,\\ \mathrm{ARL}^{-}(u) &= 1+\mathrm{ARL}^{-}(0)[1-F(K'+u)]\\ &\quad+\int_{0}^{h\sigma_x}\mathrm{ARL}^{-}(x)f(K'+u-x)\mathrm{d}x,\end{aligned}$$

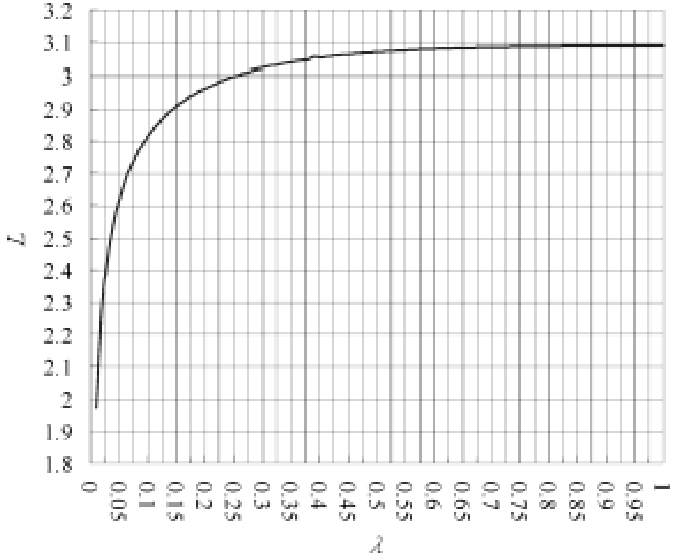
Page(1954)對於CUSUM ARL計算，利用第一步分析(first-step analysis)去建構integral equation，而Lorenzen & Vance(1986)利用其去計算upper-sided ARL $^{+}$和lower-sided ARL $^{-}$。由Kemp’s (1961)倒數規則(reciprocal rule)可獲得$(\mathrm{ARL})^{-1}=(\mathrm{ARL}^{+})^{-1}+(\mathrm{ARL}^{-})^{-1}$

$$\mathrm{ARL}(u)=1+\frac{1}{\lambda}\int_{-L\sigma_x}^{L\sigma_x}\mathrm{ARL}(x)f\left(\frac{x-(1-\lambda)u}{\lambda}\right)\mathrm{d}x.$$

上述integral equation為Fredholm integral equation of second kind為Crowder(1987)所推導出。

Control chart design under random shifts (1/5)

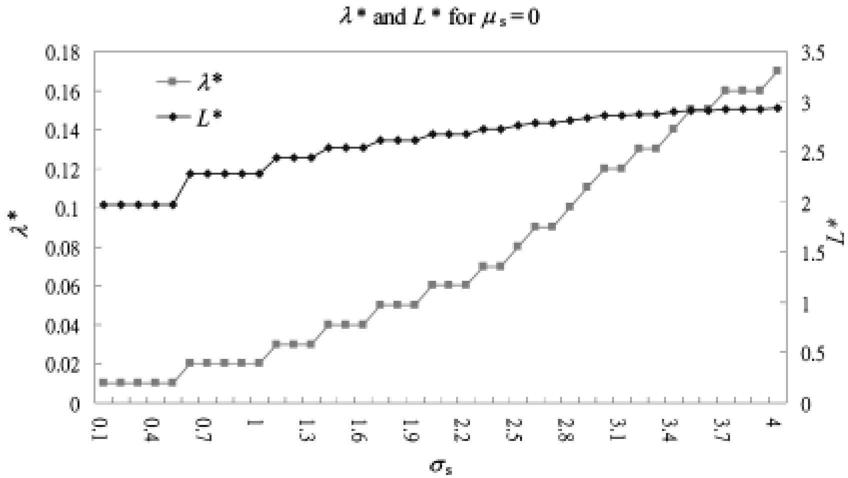
- 假設管制圖設計符合下列設定去作評估：
  1. $\text{ARL}_0 = 500$
  2. $X_i \overset{\text{iid}}{\sim} N(\mu_x,\ 1 \mid \mu_x = 0 + S)$
  3. $S \sim N(\mu_s, \sigma_s^2)$
  4. A quadratic loss function in with ω is set to one without loss of generality.

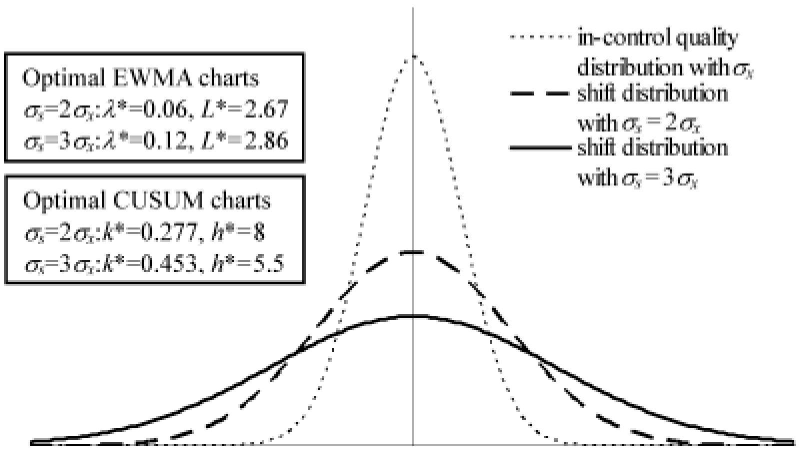


Control chart design under random shifts (2/5)

EWMA control chart design under random shifts

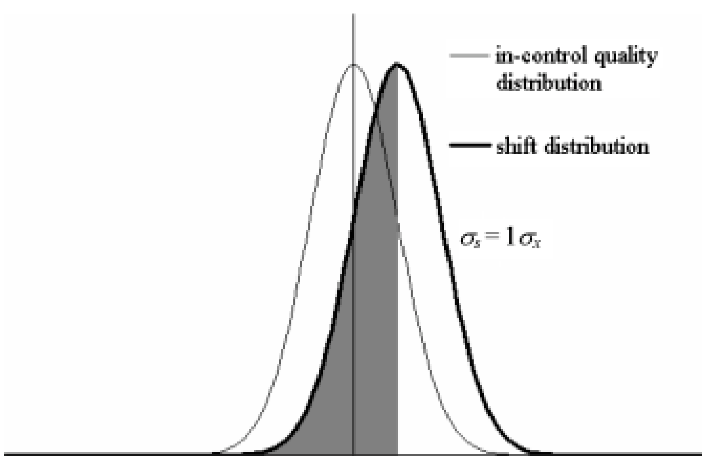
- For $\mu_s = 0$ and $\sigma_s \neq 0$ case：
  - 圖二可看出 λ^*和L^*隨著偏移大小變異數增加而遞增，其因較大偏移幅度，大偏移發生機遇亦較高。
  - 圖三在$\sigma_s = 2$和$3\sigma_x$之偏移分配試圖去給讀者有關不確定偏移幅度與管制內品質分配比較。對於變動偏移最佳設計相等於在固定偏移大小 $0.25\,\sigma_x$、$0.6\,\sigma_x$和σ_x下之傳統設計。

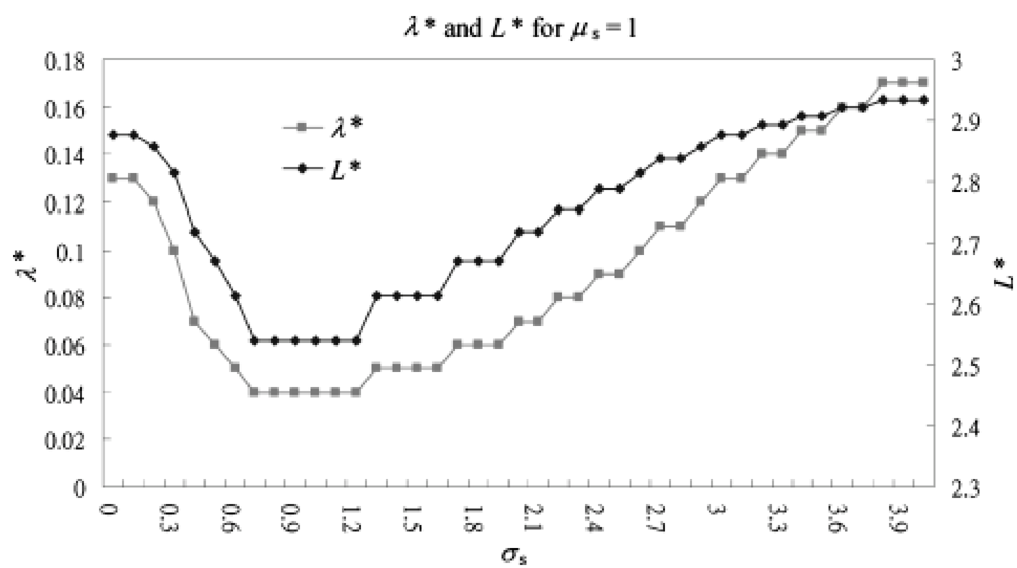




Control chart design under random shifts (3/5)

- For $\mu_s = 1$ and $\sigma_s \neq 0$ case：
  - 圖四和圖五可看出因半數不確定偏移發生在目標值附近(陰影部分)，造成一開始λ^*和L^*同時遞減，然而當不確定持續增加，由於廣泛偏移涵蓋目標值兩邊，故最佳值λ^*和L^*再次上升。

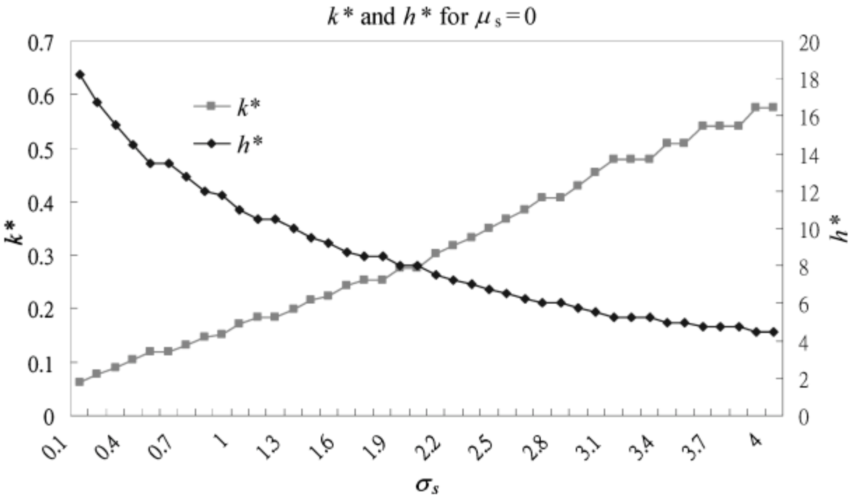


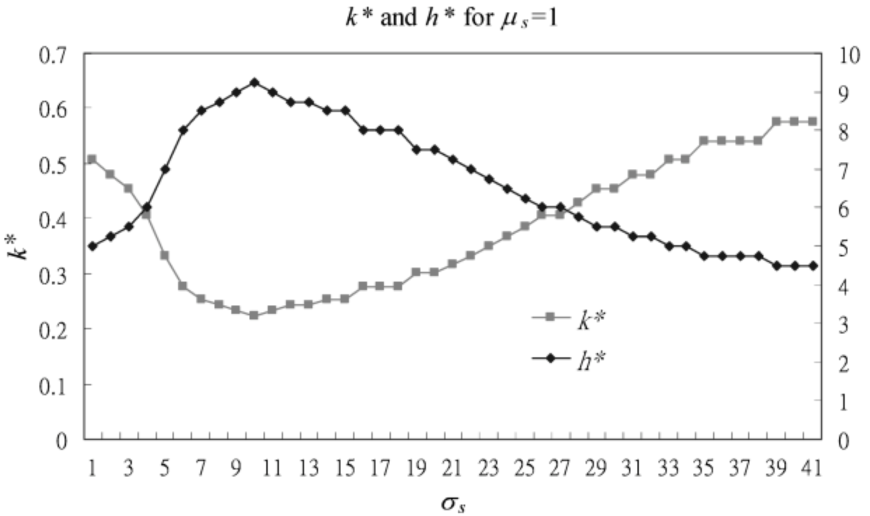


Control chart design under random shifts (4/5)

CUSUM control chart design under random shifts

- For $\mu_s = 0$ and $\sigma_s \neq 0$ case：
  - 圖七可看出隨著偏移大小變異數增加，k^*增加而h^*減少。對於變動偏移最佳設計相等於在固定偏移大小0.37 σ_x、0.56 σ_x和0.9σ_x下之傳統設計。
- For $\mu_s = 1$ and $\sigma_s \neq 0$ case：
  - 當偏移大小為不確定性時，一開始k^*遞減而h^*遞增，直到廣泛偏移涵蓋目標值兩邊，k^*才遞增而h^*才遞減。





Control chart design under random shifts (5/5)

Table 1. Optimum λ^* for EWMA chart design with $ARL_0 = 500$ under random shifts

Table 2. Optimum k^* for CUSUM chart design with $ARL_0 = 500$ under random shifts

	μ_s																
σ_s	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4
0		0.12	0.25	0.37	0.51	0.61	0.76	0.89	0.97	1.18	1.31	1.31	1.47	1.66	1.66	1.87	2.1
0.1	0.06	0.11	0.22	0.37	0.48	0.61	0.76	0.89	0.97	1.18	1.31	1.31	1.47	1.66	1.66	1.87	2.1
0.2	0.08	0.1	0.18	0.32	0.45	0.61	0.7	0.89	0.97	1.06	1.18	1.31	1.47	1.66	1.66	1.87	1.87
0.3	0.09	0.11	0.15	0.25	0.41	0.54	0.7	0.82	0.97	1.06	1.18	1.31	1.47	1.66	1.66	1.87	1.87
0.4	0.1	0.12	0.15	0.22	0.33	0.48	0.66	0.82	0.97	1.06	1.18	1.31	1.47	1.47	1.66	1.87	1.87
0.5	0.12	0.12	0.15	0.2	0.28	0.41	0.58	0.7	0.89	1.06	1.18	1.31	1.47	1.47	1.66	1.66	1.87
0.6	0.12	0.13	0.15	0.18	0.25	0.37	0.51	0.66	0.82	0.97	1.18	1.31	1.31	1.47	1.66	1.66	1.87
0.7	0.13	0.14	0.15	0.18	0.24	0.32	0.43	0.58	0.76	0.89	1.06	1.18	1.31	1.47	1.47	1.66	1.66
0.8	0.15	0.15	0.16	0.18	0.23	0.3	0.41	0.51	0.66	0.82	0.97	1.18	1.31	1.31	1.47	1.66	1.66
0.9	0.15	0.16	0.18	0.2	0.22	0.28	0.37	0.48	0.61	0.76	0.97	1.06	1.18	1.31	1.47	1.47	1.66
1	**0.17**	0.17	0.18	0.2	0.23	0.28	0.35	0.43	0.54	0.7	0.89	0.97	1.18	1.18	1.31	1.47	1.47
1.1	0.18	0.18	0.18	0.21	0.24	0.28	0.33	0.41	0.51	0.66	0.82	0.97	1.06	1.18	1.31	1.31	1.47
1.2	0.18	0.2	0.2	0.22	0.24	0.28	0.33	0.41	0.48	0.61	0.76	0.89	0.97	1.06	1.18	1.31	1.31
1.3	0.2	0.2	0.22	0.22	0.25	0.28	0.33	0.39	0.45	0.58	0.7	0.82	0.97	1.06	1.18	1.18	1.31
1.4	0.22	0.22	0.22	0.24	0.25	0.29	0.33	0.39	0.45	0.54	0.66	0.82	0.89	0.97	1.06	1.18	1.18
1.5	0.22	0.22	0.23	0.25	0.28	0.3	0.33	0.39	0.43	0.54	0.66	0.76	0.82	0.97	1.06	1.06	1.18
1.6	0.24	0.24	0.24	0.25	0.28	0.3	0.33	0.39	0.43	0.54	0.61	0.7	0.82	0.89	0.97	1.06	1.06
1.7	0.25	0.25	0.25	0.28	0.28	0.3	0.35	0.39	0.43	0.51	0.61	0.7	0.76	0.89	0.89	0.97	1.06
1.8	0.25	0.26	0.28	0.28	0.3	0.32	0.35	0.39	0.45	0.51	0.61	0.66	0.76	0.82	0.89	0.97	0.97
1.9	0.28	0.28	0.28	0.3	0.3	0.33	0.37	0.41	0.45	0.51	0.58	0.66	0.7	0.82	0.82	0.89	0.97
2	**0.28**	0.29	0.3	0.3	0.32	0.35	0.37	0.41	0.45	0.54	0.58	0.66	0.7	0.76	0.82	0.89	0.97
2.1	0.3	0.3	0.3	0.32	0.33	0.35	0.39	0.43	0.48	0.54	0.58	0.66	0.7	0.76	0.82	0.82	0.89
2.2	0.32	0.32	0.32	0.33	0.35	0.37	0.41	0.43	0.48	0.54	0.58	0.61	0.7	0.7	0.76	0.82	0.89
2.3	0.33	0.33	0.33	0.35	0.37	0.39	0.41	0.45	0.48	0.54	0.58	0.61	0.66	0.7	0.76	0.82	0.82
2.4	0.35	0.35	0.35	0.37	0.39	0.41	0.43	0.45	0.51	0.54	0.58	0.61	0.66	0.7	0.76	0.76	0.82
2.5	0.37	0.37	0.37	0.39	0.41	0.41	0.45	0.48	0.51	0.54	0.58	0.61	0.66	0.7	0.7	0.76	0.82
2.6	0.39	0.39	0.39	0.41	0.41	0.43	0.45	0.48	0.51	0.54	0.58	0.61	0.66	0.7	0.7	0.76	0.76
2.7	0.41	0.41	0.41	0.41	0.43	0.45	0.48	0.48	0.54	0.54	0.58	0.61	0.66	0.66	0.7	0.76	0.76
2.8	0.41	0.41	0.43	0.43	0.45	0.48	0.48	0.51	0.54	0.58	0.58	0.61	0.66	0.66	0.7	0.7	0.76
2.9	0.43	0.43	0.43	0.45	0.45	0.48	0.48	0.51	0.54	0.58	0.58	0.61	0.66	0.66	0.7	0.7	0.76
3	**0.45**	0.45	0.45	0.48	0.48	0.48	0.51	0.54	0.54	0.58	0.58	0.61	0.61	0.66	0.7	0.7	0.7
3.1	0.48	0.48	0.48	0.48	0.48	0.51	0.51	0.54	0.54	0.58	0.58	0.61	0.61	0.66	0.66	0.7	0.7
3.2	0.48	0.48	0.48	0.48	0.51	0.51	0.54	0.54	0.58	0.58	0.58	0.61	0.61	0.66	0.66	0.7	0.7
3.3	0.48	0.48	0.51	0.51	0.51	0.54	0.54	0.54	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.7	0.7
3.4	0.51	0.51	0.51	0.51	0.54	0.54	0.54	0.58	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.7	0.7
3.5	0.51	0.51	0.54	0.54	0.54	0.54	0.54	0.58	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.66	0.7
3.6	0.54	0.54	0.54	0.54	0.54	0.54	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.66	0.7
3.7	0.54	0.54	0.54	0.54	0.54	0.58	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.66	0.7
3.8	0.54	0.54	0.54	0.54	0.58	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.61	0.66	0.66	0.66	0.7
3.9	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.61	0.66	0.66	0.66	0.66
4	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.61	0.61	0.66	0.66	0.66	0.66

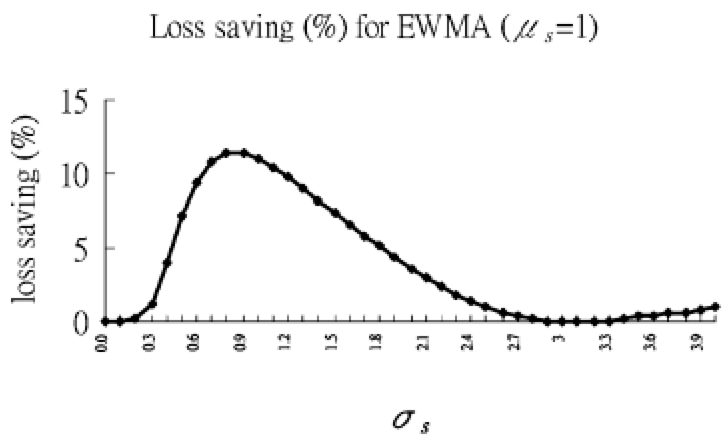
4	0.17	0.17	0.17	0.17	0.17	0.17	0.18	0.18	0.19	0.19	0.19	0.2	0.21	0.21	0.22	0.22	0.23

Quality impact reduced by designs considering random shift

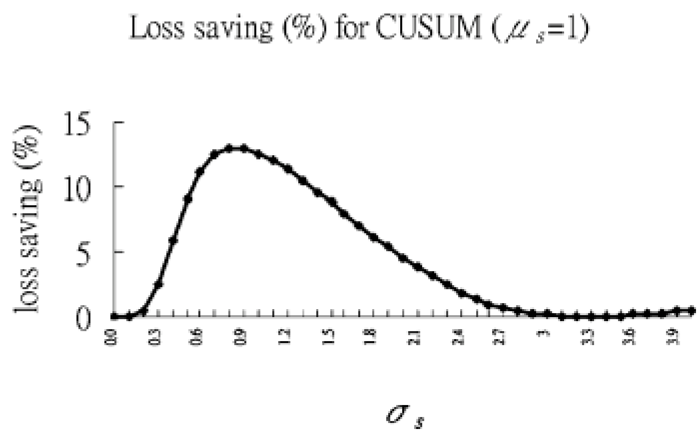
$$Loss\ saving\ (\%) = \frac{E'(QL_1) - E^*(QL_1)}{E'(QL_1)} \times 100\%$$

$E'(QL_1)$：假設$\sigma_s = 0$下，傳統最佳管制圖設計所招致品質損失

$E^*(QL_1)$：在給定σ_s已知下，所提出最佳管制圖設計所招致品質損失



(a)



(b)

- 由圖九可看出*Loss saving(%)*具有較佳表現，發生在$\sigma_s = \mu_s$附近。

Chart implementation and best designs for random shifts (1/4)

- 對於偏移如何變動無事前認識時，完成所提出管制圖設計為困難的，一般建議使用者設計從$\mu_s = 0$，其暗示變動偏移大小為在目標值中心附近，偏移大小越大發生機遇越小。

Appendix B

Table A1. Worksheet for selection of a starting EWMA control chart design

EWMA designs			*Weights*																*Weight sum*
			1	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*8*
σs	λ	L	$\mu = 0.25$	$\mu = 0.50$	$\mu = 0.75$	$\mu = 1.00$	$\mu = 1.25$	$\mu = 1.50$	$\mu = 1.75$	$\mu = 2.00$	$\mu = 2.25$	$\mu = 2.50$	$\mu = 2.75$	$\mu = 3.00$	$\mu = 3.25$	$\mu = 3.50$	$\mu = 3.75$	$\mu = 4.00$	*avg. quality impact*
0.1–0.5	0.01	1.973	842.4917	420.3798	337.6636	317.2988	322.6838	339.5717	363.671	392.0474	423.692	457.6055	493.4807	530.8546	569.5651	609.3433	650.556	693.4773	
0.6–1	0.02	2.278	841.2441	382.2891	297.2274	274.7251	276.8683	289.7139	309.1692	332.5155	358.8301	387.2091	417.3883	448.9988	481.974	515.835	550.031	584.5741	
1.1–1.3	0.03	2.4371	870.2137	366.5356	276.7065	251.9636	251.8724	262.2227	278.9318	299.3541	322.6032	347.8247	374.7552	402.943	432.1823	462.3785	494.2644	528.5169	
1.4–1.6	0.04	2.5405	910.9161	360.39	264.6255	237.5598	235.6508	244.1577	258.923	277.3131	298.4483	321.4815	346.1344	372.0356	399.3075	428.1388	458.7843	490.3104	
1.7–1.9	0.05	2.6151	957.0835	359.5726	257.1361	227.6671	224.1546	231.1657	244.4184	261.2541	280.7865	302.1813	325.2143	349.6294	375.5876	402.8745	430.8241	458.0903	
2–2.2	0.06	2.6723	1005.879	362.1513	252.5143	220.5618	215.5672	221.2926	233.2966	248.8737	267.1302	287.2516	309.0568	332.2777	356.848	382.1558	407.2737	431.4779	
2.3–2.4	0.07	2.718	1055.902	367.1144	249.8622	215.344	208.9384	213.5144	224.4457	238.9644	256.1669	275.2458	296.0109	318.0993	341.2119	364.6302	387.72	410.5772	
2.5–2.7	0.08	2.7556	1106.379	373.8646	248.6559	211.4874	203.7116	207.2313	217.2135	230.8151	247.1147	265.2927	285.1122	306.1122	327.8821	349.8358	371.8394	394.5919	
2.6–2.7	0.09	2.7872	1156.803	382.0119	248.5611	208.6584	199.5339	202.0622	211.1848	223.9714	239.4729	256.8427	275.7881	295.79	316.4535	337.435	359.0765	382.3756	
2.8	0.1	2.8143	1207.087	391.3302	249.3725	206.6463	196.1794	197.7612	206.0904	218.1373	232.9171	249.5493	267.6964	286.8294	306.6545	327.0813	348.8009	372.8535	
2.9	0.11	2.8378	1256.909	401.6096	250.9254	205.2897	193.4778	194.1449	201.7304	213.0948	227.2119	243.1675	260.5991	279.0048	298.2324	318.4008	340.4015	365.1413	
3–3.1	**0.12**	**2.8583**	**1305.932**	**412.6713**	**253.0905**	**204.4666**	**191.3024**	**191.0773**	**197.9569**	**208.6836**	**222.1868**	**237.5225**	**254.3223**	**272.1352**	**290.9549**	**311.0451**	**333.3639**	**358.5813**	
3.2–3.3	0.13	2.8765	1354.461	424.4821	255.8167	204.1152	189.582	188.4779	194.6807	204.8058	217.7374	232.5068	248.7552	266.0913	284.6381	304.7358	327.3059	352.734	
3.4	0.14	2.8928	1402.522	436.971	259.0447	204.1752	188.2512	186.2752	191.8233	201.3765	213.7741	228.0265	243.7952	260.747	279.1054	299.229	321.929	347.2984	
3.5–3.6	0.15	2.9073	1449.525	449.9698	262.6841	204.5739	187.2394	184.3965	189.3077	198.3144	210.2108	223.9905	239.3399	255.9759	274.1906	294.3148	317.0028	342.0715	
3.7–3.9	0.16	2.9204	1495.861	463.4995	266.7278	205.2927	186.5212	182.8105	187.0984	195.5796	207.0043	220.3502	235.3297	251.6977	269.7838	289.857	312.3902	336.9565	
4	0.17	2.9323	1541.554	477.524	271.1467	206.3031	186.0653	181.483	185.1574	193.131	204.1095	217.0549	231.703	247.8332	265.785	285.743	307.992	331.8992	

Chart implementation and best designs for random shifts (2/4)

Table A2. Worksheet for selection of a starting CUSUM control chart design

CUSUM designs			*Weights*																*Weight sum 8 avg. quality impact*
			1	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	
σ_s	*k*	*h*	$\mu = 0.25$	$\mu = 0.50$	$\mu = 0.75$	$\mu = 1.00$	$\mu = 1.25$	$\mu = 1.50$	$\mu = 1.75$	$\mu = 2.00$	$\mu = 2.25$	$\mu = 2.50$	$\mu = 2.75$	$\mu = 3.00$	$\mu = 3.25$	$\mu = 3.50$	$\mu = 3.75$	$\mu = 4.00$	
0.2	0.0762	16.75	966.3902	494.0494	400.3978	377.6591	384.5266	404.6739	433.1368	466.5114	503.6143	543.2868	585.1583	628.6964	673.7685	720.1418	767.8462	816.2638	
0.3	0.0904	15.5	950.646	471.7524	378.9359	355.8378	361.4247	379.7751	406.0867	437.0922	471.6629	508.6941	547.8356	588.5869	630.8607	674.3544	718.7678	763.876	
0.4	0.1037	14.5	942.8321	454.7228	362.1036	338.5784	343.0858	359.9693	384.5421	413.6404	446.1771	481.0889	518.0455	556.5732	596.5429	637.5689	679.651	723.264	
0.5–0.6	0.119	13.5	939.7678	438.4808	345.5762	321.4867	324.862	340.2509	363.0689	390.2499	420.7441	453.5317	488.298	524.5688	562.1933	600.9704	641.2752	683.0734	
0.7	0.1322	12.75	942.0752	427.1005	333.4989	308.8497	311.3223	325.5624	347.0479	372.7791	401.7345	432.9224	466.0331	500.6019	536.5573	573.8296	612.5321	651.8218	
0.8	0.1472	12	949.3905	416.61	321.7685	296.4054	297.9181	310.9787	331.1142	355.3844	382.7916	412.3709	443.8222	476.7232	511.0752	546.6884	583.1412	619.4875	
0.9	0.1526	11.75	952.7945	413.2884	317.9194	292.2881	293.4694	306.1311	325.8134	349.5943	376.4837	405.5256	436.4264	468.7801	502.5843	537.5609	573.1728	608.6808	
1	0.1705	11	968.234	404.2957	306.7492	280.1462	280.2707	291.7034	310.0069	332.307	357.6348	385.0608	414.3164	445.016	477.0467	509.9096	543.2057	577.2238	
1.1–1.2	0.1838	10.5	982.0535	399.002	299.5477	272.1747	271.5508	282.1418	299.5133	320.818	345.0996	371.4473	399.6019	429.1547	459.8987	491.3701	523.6233	557.4841	
1.3	0.1986	10	1000.602	394.6473	292.6964	264.3919	262.9598	272.6787	289.1011	309.3991	332.6271	357.8906	384.9238	413.2835	442.7417	473.0457	504.7247	538.6243	
1.4	0.215	9.5	1023.835	391.2947	286.2006	256.7904	254.4876	263.3032	278.7588	298.0386	320.2049	344.3723	370.2598	397.4131	425.6977	455.1312	486.4376	519.9149	
1.5	0.2238	9.25	1036.895	389.976	283.072	253.0449	250.2849	258.6381	273.6043	292.371	314.003	337.6172	362.9267	389.4904	417.2534	446.3325	477.3852	510.3005	
1.6	0.2431	8.75	1068.645	388.5864	277.2653	245.7874	242.0351	249.4241	263.3899	281.1159	301.6669	324.1634	348.3218	373.764	400.5947	428.9604	459.0801	490.043	
1.7–1.8	0.2536	8.5	1086.982	388.5062	274.575	242.2625	237.9759	244.8643	258.3194	275.5179	295.5229	317.4577	341.0507	365.9647	392.3522	420.2765	449.647	479.2803	
1.9–2	0.2766	8	1129.62	389.9015	269.7371	235.4761	230.024	235.8638	248.2713	264.3973	283.2996	304.1121	326.6027	350.4957	375.9174	402.6218	429.9482	456.6248	
2.1	0.3027	7.5	1181.535	393.8786	265.7987	229.1198	222.3406	227.0535	238.3707	253.3974	271.1839	290.8798	312.2853	335.1146	359.3103	384.3021	409.2134	433.2965	
2.2	0.3171	7.25	1211.485	397.0658	264.2487	226.1388	218.6201	222.7334	233.4861	247.9514	265.1747	284.3111	305.163	327.4073	350.8676	374.88	398.6506	421.8401	
2.3	0.3325	7	1244.247	401.1821	263.0245	223.3084	214.9906	218.476	228.6491	242.5442	259.1995	277.7708	298.0497	319.6609	342.319	365.3404	388.1332	410.7899	
2.4	0.349	6.75	1280.047	406.3604	262.1758	220.6501	211.4637	214.2885	223.8652	237.1799	253.2601	271.256	290.9353	311.8651	333.6837	355.7677	377.8173	400.3201	
2.5	0.3667	6.5	1318.933	412.7365	261.7554	218.1856	208.051	210.1782	219.139	231.8612	247.3563	264.7618	283.8111	304.0216	325.004	346.2677	367.8529	390.5515	
2.6	0.3858	6.25	1361.843	420.5993	261.875	215.968	204.785	206.1676	214.4881	226.6018	241.4977	258.2932	276.6841	296.1567	316.3519	336.9642	358.37	381.5365	
2.7–2.8	0.4064	6	1408.557	430.097	262.5967	214.0216	201.6761	202.2616	209.9141	221.4008	235.6808	251.8456	269.5585	288.3012	307.8011	327.9595	349.4364	373.2229	
2.9	0.4288	5.75	1460.61	441.7068	264.1074	212.4348	198.7782	198.4973	205.4455	216.2808	229.9242	245.4387	262.467	280.5165	299.4436	319.3434	341.0638	365.4683	
3	**0.4531**	**5.5**	**1517.337**	**455.587**	**266.4881**	**211.2381**	**196.1024**	**194.8781**	**201.0812**	**211.2384**	**224.2246**	**239.0753**	**255.4295**	**272.8478**	**291.336**	**311.1361**	**333.1602**	**358.0124**	
3.1–3.3	0.4795	5.25	1578.697	472.0485	269.8873	210.4977	193.6826	191.4226	196.8333	206.2818	218.5913	232.7723	248.4797	265.3432	283.5116	303.3014	325.554	350.5204	
3.4–3.5	0.5085	5	1647.404	491.9845	274.6719	210.3871	191.6213	188.2003	192.7547	201.4558	213.0676	226.5787	241.6753	258.0574	275.982	295.7525	318.0172	342.6314	
3.6–3.8	0.5402	4.75	1721.377	515.5115	280.967	210.9608	189.9374	185.2151	188.8427	196.7561	207.6529	220.5007	235.0239	250.9775	268.6698	288.3059	310.2483	333.9741	
3.9–4	0.5751	4.5	1802.138	543.5187	289.1993	212.4244	188.7459	182.539	185.1502	192.2264	202.3877	214.5761	228.5491	244.0862	261.4777	280.7626	301.9722	324.2791	

Chart implementation and best designs for random shifts (3/4)

- 文獻中，CUSUM程序用於檢測一固定偏移已被證實，進一步，Moustakides(1986)對CUSUM管制圖去計算最小化條件平均延遲時間，其最小值確實等於ARL1。然而，Stivastava & Wu(1993)指出，等式並不成立於EWMA管制圖，故去比較CUSUM與EWMA管制圖間ARL1績效不確切，直到Lucas & Saccuci(1990) 作出對於較小固定偏移EWMA管制圖績效相對較好。

Chart implementation and best designs for random shifts (4/4)

Table 3. Least quality impacts under $ARL_0 = 500$ and random shifts

σ_s	μ_s: 0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4
0	0	841.24	359.57	248.56	204.12	185.84	176.99	172.30	170.76	171.55	173.34	176.07	179.19	183.41	188.82	195.56	204.55
0.1	1189.73	953.15	388.97	254.37	206.99	186.95	177.98	172.80	171.19	171.83	173.63	176.22	179.37	183.56	188.99	195.73	204.81
0.2	1136.53	905.78	495.73	285.84	217.13	191.55	180.96	174.77	172.45	172.77	174.40	176.76	179.87	184.10	189.48	196.34	205.47
0.3	979.29	848.68	576.33	356.65	246.04	202.66	186.31	178.33	174.88	174.41	175.60	177.72	180.76	185.06	190.33	197.38	206.52
0.4	859.39	785.55	607.07	426.17	298.01	228.79	198.18	185.59	179.29	177.17	177.53	179.23	182.14	186.46	191.60	198.88	208.00
0.5	772.18	727.70	607.83	470.63	352.40	269.63	221.85	198.87	187.16	182.08	180.69	181.58	184.21	188.02	193.36	200.70	209.94
0.6	705.17	674.19	594.57	493.46	394.57	313.12	255.50	220.45	201.93	190.75	186.29	185.42	187.00	190.26	195.75	202.86	212.38
0.7	651.13	630.74	576.49	501.90	422.57	350.24	291.43	249.02	222.53	205.52	195.23	191.14	190.89	193.61	198.62	205.66	215.14
0.8	609.65	595.76	557.97	501.95	439.51	378.28	323.83	279.63	247.54	226.42	209.70	200.65	197.50	198.48	202.33	209.34	218.37
0.9	577.37	567.68	540.23	497.66	448.78	398.40	350.27	308.10	274.21	248.96	228.89	214.31	207.02	205.19	207.88	213.59	222.49
1	552.05	544.88	523.09	491.46	453.01	411.93	370.83	332.60	299.65	272.76	251.29	232.23	220.41	215.34	215.14	219.54	227.44
1.1	530.00	524.34	508.35	484.76	454.15	420.79	386.16	352.69	322.32	295.82	272.78	252.42	236.57	227.76	225.12	227.40	233.78
1.2	511.89	507.74	495.94	477.40	453.54	426.44	397.51	368.82	341.72	316.87	293.81	273.48	255.29	243.26	237.39	237.23	242.00
1.3	497.44	494.39	485.54	470.37	451.84	429.85	405.86	381.55	357.94	335.39	313.37	292.75	274.68	260.12	251.91	249.30	251.70
1.4	485.81	483.19	475.67	464.22	449.25	431.58	411.97	391.67	371.42	351.33	330.90	310.90	293.25	278.01	267.55	262.81	263.44
1.5	475.18	473.23	467.62	459.03	446.69	432.42	416.49	399.69	382.48	364.84	346.27	327.45	310.18	295.95	284.25	277.58	276.03
1.6	466.76	465.32	461.17	453.79	444.40	432.81	419.92	406.09	391.61	376.28	359.56	342.25	325.83	311.81	300.50	292.73	289.83
1.7	460.10	458.84	455.19	449.54	442.14	433.04	422.65	411.25	399.13	385.81	370.95	355.25	340.05	326.63	315.98	307.91	303.51
1.8	453.96	453.04	450.37	446.26	440.10	432.99	424.72	415.48	405.33	393.77	380.62	366.58	352.71	340.18	329.87	322.47	317.44
1.9	449.29	448.63	446.72	443.12	438.63	432.94	426.41	419.01	410.44	400.40	388.82	376.34	363.87	352.36	342.61	335.24	330.36
2	445.56	444.99	443.35	440.83	437.41	433.12	427.95	421.82	414.54	405.86	395.74	384.71	373.62	363.19	354.11	346.94	342.05
2.1	442.45	442.08	440.99	439.17	436.46	433.18	429.09	424.14	417.93	410.35	401.53	391.90	382.06	372.66	364.30	357.46	352.47
2.2	440.39	440.12	439.19	437.77	435.96	433.39	430.15	425.96	420.62	414.04	406.37	397.93	389.26	380.86	373.22	366.75	361.75
2.3	438.58	438.39	437.85	436.98	435.46	433.51	430.89	427.43	422.76	417.05	410.37	403.00	395.38	387.86	380.89	374.82	369.89
2.4	437.54	437.45	436.97	436.24	435.22	433.73	431.53	428.42	424.48	419.49	413.65	407.23	400.49	393.78	387.44	381.75	376.89
2.5	436.64	436.56	436.33	435.82	435.00	433.72	431.86	429.23	425.72	421.35	416.31	410.67	404.69	398.72	392.92	387.57	382.81
2.6	436.13	436.05	435.81	435.47	434.76	433.69	432.01	429.68	426.68	422.80	418.39	413.42	408.10	402.73	397.40	392.36	387.74
2.7	435.67	435.64	435.49	435.10	434.55	433.58	432.07	430.03	427.25	423.88	419.97	415.57	410.81	405.96	401.04	396.25	391.74
2.8	435.26	435.21	435.10	434.77	434.22	433.24	431.89	430.06	427.61	424.64	421.13	417.19	412.90	408.46	403.88	399.33	394.88
2.9	434.87	434.82	434.65	434.37	433.74	432.90	431.70	429.99	427.76	425.04	421.90	418.34	414.44	410.31	406.03	401.65	397.31
3	434.43	434.36	434.15	433.81	433.29	432.45	431.29	429.68	427.65	425.19	422.34	419.08	415.48	411.62	407.54	403.35	399.04
3.1	433.84	433.79	433.64	433.25	432.71	431.92	430.81	429.31	427.40	425.12	422.48	419.45	416.09	412.44	408.53	404.44	400.20
3.2	433.23	433.17	432.97	432.65	432.09	431.31	430.17	428.75	427.00	424.86	422.37	419.50	416.31	412.80	409.07	405.06	400.84
3.3	432.61	432.56	432.33	431.93	431.40	430.54	429.51	428.09	426.39	424.39	422.02	419.27	416.20	412.80	409.17	405.23	401.06
3.4	431.84	431.77	431.56	431.17	430.54	429.76	428.66	427.35	425.66	423.72	421.42	418.79	415.80	412.48	408.92	405.02	400.85
3.5	431.02	430.94	430.69	430.27	429.70	428.85	427.78	426.44	424.83	422.89	420.64	418.08	415.15	411.89	408.37	404.50	400.34
3.6	430.10	430.02	429.78	429.37	428.71	427.87	426.81	425.45	423.87	421.94	419.70	417.17	414.29	411.05	407.56	403.71	399.55
3.7	429.14	429.04	428.77	428.31	427.68	426.85	425.71	424.39	422.76	420.87	418.61	416.09	413.23	410.01	406.53	402.69	398.51
3.8	428.05	427.96	427.69	427.25	426.58	425.67	424.56	423.22	421.56	419.64	417.40	414.86	412.01	408.78	405.31	401.47	397.26
3.9	426.96	426.87	426.56	426.06	425.36	424.45	423.35	421.92	420.27	418.30	416.06	413.50	410.64	407.41	403.93	400.07	395.85
4	425.70	425.60	425.29	424.79	424.09	423.19	421.98	420.55	418.87	416.87	414.63	412.02	409.14	405.90	402.41	398.52	394.29

Note: Shaded area: least quality impacts by CUSUM; Clear area: least quality impacts by EWMA.

Conclusons

- 本研究為第一篇對於不確定性偏移大小且具有不同品質損失水準去設計EWMA和CUSUM管制圖文獻，它發現當偏移大小為不確定所設計出最佳管制圖為更加保守。
- 對於經驗資淺(naïve)使用者，建議EWMA管制圖設計為$\mu_s = 0$ 和$\sigma_s = 3$，參數為$\lambda^* = 0.12$和$L^* = 2.86$。