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Economic-statistical Design of 2-of-2 and 2-of-3 Run Rule Scheme

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INTRODUCTION

作者	年代	相關性
Wheeler	1983	合併多項規則於單邊Shewhart管制圖中，並提供達10個連串長度程度之機率精確算式。
Duncan	1986	將規則單獨運用於各獨立假設下，去指示管制外訊號之整體誤警率經驗式。
Khoo & Ariffin	2006	提出兩改善連串規則去增進Klein所建議兩規則(alternative rules)績效。
Acosta-Mejia	2007	分析具有k-of-k和k-of-(k+1)連串規則之統計特性，以支援Shewhart管制圖。

DEVELOPMENT OF MARKOV CHAIN APPROACH (1/5)

Markov chain approach

- Average length of renewal cycle; ARLC: 從管制內製程開始到真實管制外訊號產生狀態之平均觀察數目。
  - 經驗式：$h \times$ ARLC
- Klein利用Markov chain方式推導出2-of-2和2-of-3連串規則計劃ARL，但無法直接公式化ARLC，因為在Klein的狀態定義中並無思考生產製程是否為管制內。

Models

Model 1 traditional Shewhart control chart scheme
Model 2 2-of-2 runs rule scheme
Model 3 2-of-3 runs rule scheme

Assumptions

(1) The in-control period is an exponential random variable with mean $1/\lambda$.
(2) The process starts anew after each false alarm.
(3) There is only one assignable cause and a shift of process average occurs by a known amount.

DEVELOPMENT OF MARKOV CHAIN APPROACH (2/5)

State	Interpretation
$\{O,I\}$	A point is between LCL and UCL during in-control period
$\{U,I\}$	A point is above the UCL during in-control period
$\{L,I\}$	A point is below the LCL during in-control period
$\{S,I\}$	Two successive points are either above the UCL or below the LCL during in-control period
$\{O,N\}$	A point is between LCL and UCL during out-of-control period
$\{U,N\}$	A point is above the UCL during out-of-control period
$\{L,N\}$	A point is below the LCL during out-of-control period
$\{S,N\}$	Two successive points are either above the UCL or below the LCL during out-of-control period, the absorbing state

p

$p_U = 1-\Phi(k_2)$

$p_L = \Phi(-k_2),\ p + p_U + p_L = 1$

q

$q_U = 1-\Phi(k_2 - \delta\sqrt{n})$

$q_L = \Phi(-k_2 - \delta\sqrt{n}),\ q + q_U + q_L = 1$

- 狀態$\{S,I\}$意謂著製程的一個錯誤警報，因為第二假設關係，故可思考成與狀態$\{O,I\}$相同。
- 定義管制內期間，可歸屬原因在連續兩抽樣之間發生機率為 $r = 1 - e^{-\lambda h}$

Table I. State transition matrix for 2-of-2 runs rule scheme

		States at $(n+1)$th sampling sequence						
		$\{O,I\}$	$\{U,I\}$	$\{L,I\}$	$\{S,I\}$	$\{O,N\}$	$\{U,N\}$	$\{L,N\}$
States at (n)th sampling sequence	$\{O,I\}$	$(1-r)p$	$(1-r)p_U$	$(1-r)p_L$	0	rq	rq_U	rq_L
	$\{U,I\}$	$(1-r)p$	0	0	$(1-r)(1-p)$	rq	0	0
	$\{L,I\}$	$(1-r)p$	0	0	$(1-r)(1-p)$	rq	0	0
	$\{S,I\}$	$(1-r)p$	$(1-r)p_U$	$(1-r)p_L$	0	rq	rq_U	rq_L
	$\{O,N\}$	0	0	0	0	q	q_U	q_L
	$\{U,N\}$	0	0	0	0	q	0	0
	$\{L,N\}$	0	0	0	0	q	0	0

- 令M為一元素ij所構成潛在矩陣(potential matrix)，即初始狀態i條件下，返回狀態j之期望數； $M = (I - Q)^{-1}$

DEVELOPMENT OF MARKOV CHAIN APPROACH (3/5)

- 管制圖ARLC等同於訊號產生或到達吸態之前，在轉移狀態下期望轉移次數；$ARLC = t'M1$

  t → [1 0 0 0 0 0 0]

$$ALRC_{\text{Model 1}} = \frac{1}{r} + \frac{q}{1-q}$$

$$ALRC_{\text{Model 2}} = \frac{1}{r} + \frac{1-q+2q_Uq_L+q(1+q_L)(1+q_U)(1+p_U-rp_U)}{(1-q_Uq_L-q(1+q_L)(1+q_U))(1+p_U-rp_U)}$$

$$ARL_{\text{Model 1}} = \frac{1}{1-p}$$

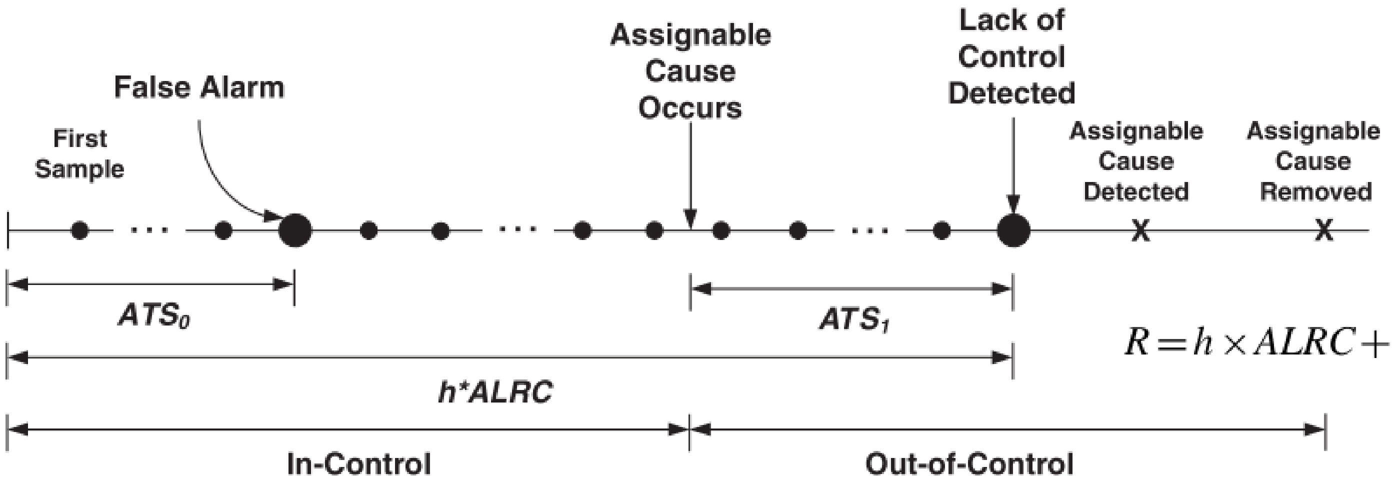
$$ARL_{\text{Model 2}} = \frac{1+p_U}{2p_U^2}$$

$$ARL_{\text{Model 3}} = \frac{1+p_U(3+p_U-2p_U^2)}{2p_U^2(2+p_U-2p_U^2)}$$

$$ATS_0 = h \times ARL$$

$$ATS_1 = h \times ALRC - \frac{1}{\lambda}$$

DEVELOPMENT OF MARKOV CHAIN APPROACH (4/5)



$$R = h \times ALRC + \frac{(1-\gamma_1)s\,T_0}{ARL} + T_1 + T_2 + nE$$

Figure 1. Diagram of in-control and out-of-control states of a process

The cycle time in this paper consists of the following three parts:

(a) the time until the chart gives a true out-of-control signal

$$A_1 = h \times ALRC + \frac{(1-\gamma_1)sT_0}{ARL},\ s = e^{-\lambda h}/(1-e^{-\lambda h})$$

(b) the time to discover the assignable cause and repair the process; $A_2 = T_1 + T_2$

(c) the time to analyze and chart the sample; $A_3 = nE$

COST MODEL (5/5)

To derive the cost function, we consider four cost elements:

(a) cost due to non-conformities produced while the process is in control as well as out of control

$$B_1 = C_0(1/\lambda) + C_1(R - 1/\lambda - (1-\gamma_1)T_0 S/ARL - (1-\gamma_1)T_1 - (1-\gamma_2)T_2)$$

(b) sampling cost

$$B_2 = (a+bn)(R - 1/\lambda - (1-\gamma_1)T_0 S/ARL - (1-\gamma_1)T_1 - (1-\gamma_2)T_2)/h$$

(c) cost for location and repair of the assignable cause ; $B_3 = W$

(d) cost due to false alarm; $B_4 = Y\left(\frac{S}{ARL}\right)$

Design parameters

- k_i control limit coefficient for Model i
- n fixed sample size
- h fixed sampling interval

Cost and time parameters

- C_0 hourly non-conforming production cost while the production process is in control
- C_1 hourly non-conforming production cost while the production process is out of control
- Y cost per false alarm
- W cost to locate and repair the assignable cause
- a fixed cost per sample
- b cost per unit sampled
- E expected time to sample and chart one item
- T_0 expected search time when false alarm has been given
- T_1 expected time to discover the assignable cause
- T_2 expected time to repair the process

ECONOMIC-STATISTICAL DESIGN

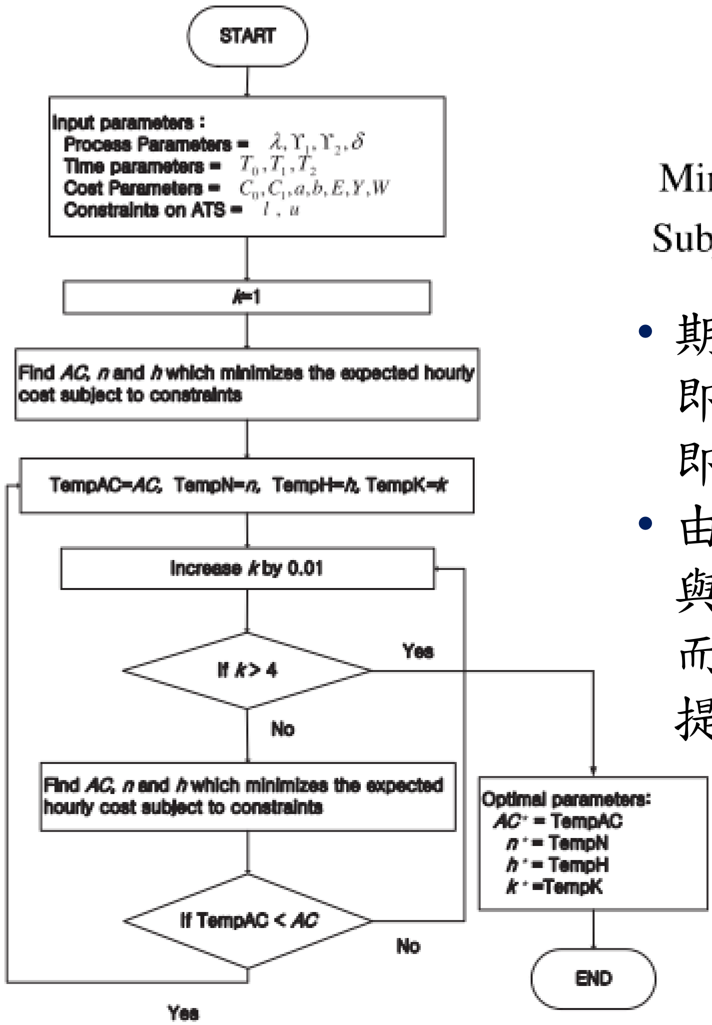


Figure 2. Constrained optimization search procedure for the economic-statistical design

$$\text{Minimize} \quad AC$$
$$\text{Subject to} \quad ATS_0 > l, \quad ATS_1 < u$$

- 期望時間成本為Renewal reward process
  即預期週期成本與預期週期時間比例
  即$AC=(B_1+B_2+B_3+B_4)/R$
- 由於再加入了統計性質限制，經濟統計設計與純經濟設計比較，可能應多付懲罰 ，然而，懲罰成本對於降低錯誤警報所得利益和提早偵測可歸屬原因發生並不顯著。

NUMERICAL ILLUSTRATIONS (1/4)

Numerical example

Table II. Statistical and economical results for the illustrative example

	n^*	h^*	k^*	ATS_0	ATS_1	AC^*	% Ratio
Model 1	16	2.64	2.79	500	2.24	225.16	
Model 2	8	0.92	1.88	526.81	2.64	222.20	1.31
Model 3	8	0.98	2.00	500	2.58	219.40	2.56

the cost parameters and the process parameters as :

λ=1/50, δ=0.86, C_0=\$114.24,

C_1=\$949.20, W=\$1086, Y=\$977.40, a=0, b=\$4.22,

γ_1=1, γ_2=0, T_0=T_1=E=5/60, T_2=45/60 (Lorenzen and Vance;1986)

NUMERICAL ILLUSTRATIONS (2/4)

Sensitivity analysis

Table III. Cost and process parameters

Parameters	Value Low	Value High
a	1	2
b	0.2	0.6
W	150	300
Y	200	500
C_0	100	200
C_1	250	500
λ	0.01	0.05
δ	0.5	1.0

Table IV. Fixed parameters and bounds of constraints

Parameters	Value
T_0	5.5
T_1	3.5
T_2	8
E	0.275
γ_1	1
γ_2	0
l	500
u	8

NUMERICAL ILLUSTRATIONS (3/4)

Table V. Design set for sensitivity analysis

Case	λ	C_0	C_1	W	a	b	Y	δ
1	0.01	100	250	150	1	0.2	200	0.5
2	0.05	100	250	150	1	0.6	500	1.0
3	0.01	200	250	150	2	0.2	500	1.0
4	0.05	200	250	150	2	0.6	200	0.5
5	0.01	100	500	150	2	0.6	500	0.5
6	0.05	100	500	150	2	0.2	200	1.0
7	0.01	200	500	150	1	0.6	200	1.0
8	0.05	200	500	150	1	0.2	500	0.5
9	0.01	100	250	300	2	0.6	200	1.0
10	0.05	100	250	300	2	0.2	500	0.5
11	0.01	200	250	300	1	0.6	500	0.5
12	0.05	200	250	300	1	0.2	200	1.0
13	0.01	100	500	300	1	0.2	500	1.0
14	0.05	100	500	300	1	0.6	200	0.5
15	0.01	200	500	300	2	0.2	200	0.5
16	0.05	200	500	300	2	0.6	500	1.0

Table Vt chart and runs rules

Model 1							Model 3						
n^*	h^*	k^*	ATS_0	ATS_1	AC^*	% Ratio	n^*	h^*	k^*	ATS_0	ATS_1	AC^*	% Ratio
24	1.69	2.93	500	4.53	115.83	1.7	14	0.98	2.00	500	4.75	113.42	2.1
9	0.94	3.11	500	1.59	118.32	1.0	7	0.61	2.10	500	1.48	116.85	1.2
11	3.34	2.87	813.26	3.30	193.48	−0.2	8	1.80	2.16	1971.48	4.01	193.09	0.2
24	1.99	2.88	500	4.99	188.81	1.2	14	0.98	2.00	500	4.76	185.70	1.6
24	1.75	2.92	500	4.61	151.46	3.4	14	0.93	2.01	500	4.60	144.97	4.3
10	0.45	3.32	500	0.80	167.46	2.4	8	0.32	2.23	500	0.75	163.03	2.6
10	1.69	2.93	500	2.02	212.83	0.3	6	0.85	2.03	500	2.27	211.84	0.5
24	0.67	3.21	500	2.64	263.56	4.1	19	0.37	2.20	500	1.58	250.62	4.9
10	2.80	2.77	500	2.90	111.30	0.3	7	1.61	1.89	500	3.36	110.75	0.5
23	0.76	3.17	500	3.09	140.48	3.4	17	0.67	2.08	500	2.83	134.71	4.1
28	6.00	2.52	511.15	7.94	200.57	−0.1	14	2.04	1.84	511.25	8.00	200.50	0.0
9	0.82	3.15	500	1.45	176.08	0.4	7	0.55	2.12	500	1.36	175.27	0.5
9	0.79	3.16	500	1.41	125.26	0.8	7	0.55	2.15	576.01	1.39	123.98	1.0
23	0.74	3.18	500	3.03	225.17	5.8	17	0.61	2.10	500	2.64	209.45	7.0
21	0.97	3.10	500	4.14	228.26	1.6	16	0.78	2.05	500	3.39	223.64	2.0
9	0.69	3.20	500	1.29	232.01	1.1	7	0.46	2.16	500	1.16	228.40	1.6

- 兩種連串規則下，所有case之ARL_0均勝過Shewhart管制圖；而ATS_1在三種model相似，最大差距不為2h。
- 連串規則計劃下相對於Shewhart管制圖計劃所需較小抽樣大小和較短抽樣區間。
- Model 3之管制界限係數大於Model 2，此近似Kelin所提出。

NUMERICAL ILLUSTRATIONS (4/4)

Table VII. Sensitivity analysis results for 2-of-2 runs rules

	n^*	h^*	k^*	AC^*
a	+			
b	−		−	
W				
Y				
C_0				+
C_1		−	+	+
λ		−	+	
δ	−			

Table VIII. Sensitivity analysis results for 2-of-3 runs rules

	n^*	h^*	k^*	AC^*
a	+			
b			−	
W				
Y				
C_0				+
C_1	−	−	+	+
λ	−	−	+	
δ	−		+	

- 期望時間成本主要因缺陷產品成本影響,而與非隨機原因發生率和標準化製程平均偏移兩者幾乎無影響。
- 製程參數λ, δ主要影響設計參數n, h和k，當然，敏感度分析結果可改變涉及到輸入參數水準。

CONCLUSIONS

- 為了達到SPC的目標一致性，經濟思考應合併於管制圖設計中，從經濟觀點上，儘管它易於在現實製造業執行連串規則計劃或無需過多成本和因素從傳統Shewhart管制圖改變連串規則計劃，仍很少把注意力放在連串規則上。