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An alternative approach to fuzzy control charts :Direct fuzzy approach

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目錄

1.Introduction

- 當品質為外觀特徵時，柔軟性或是顏色，為非數值資料時，在此情況下，使用傳統管制圖是有問題的
- 模糊理論可以將此進行分集區，方便決策

比較模糊理論管制圖與Shewhart管制圖如表1

比較問題	傳統 Shewhart 管制圖	模糊管制圖
品質特徵	只有一個品質特徵	多重品質特徵
資料	歷史資料	專家經驗資料
判斷	管制內或在管制外	經由中間語言決策
優點	1. 考慮一個簡單的品質特徵 2. 客觀	1. 由專家經驗提供更準確的控制標準 2. 使用上較靈活
缺點	1. 固定的管制界線 2. 樣本大小影響管制界線	1. 推理結果是基於專家經驗

2. Fuzzy control charts based on α-cuts

圖1

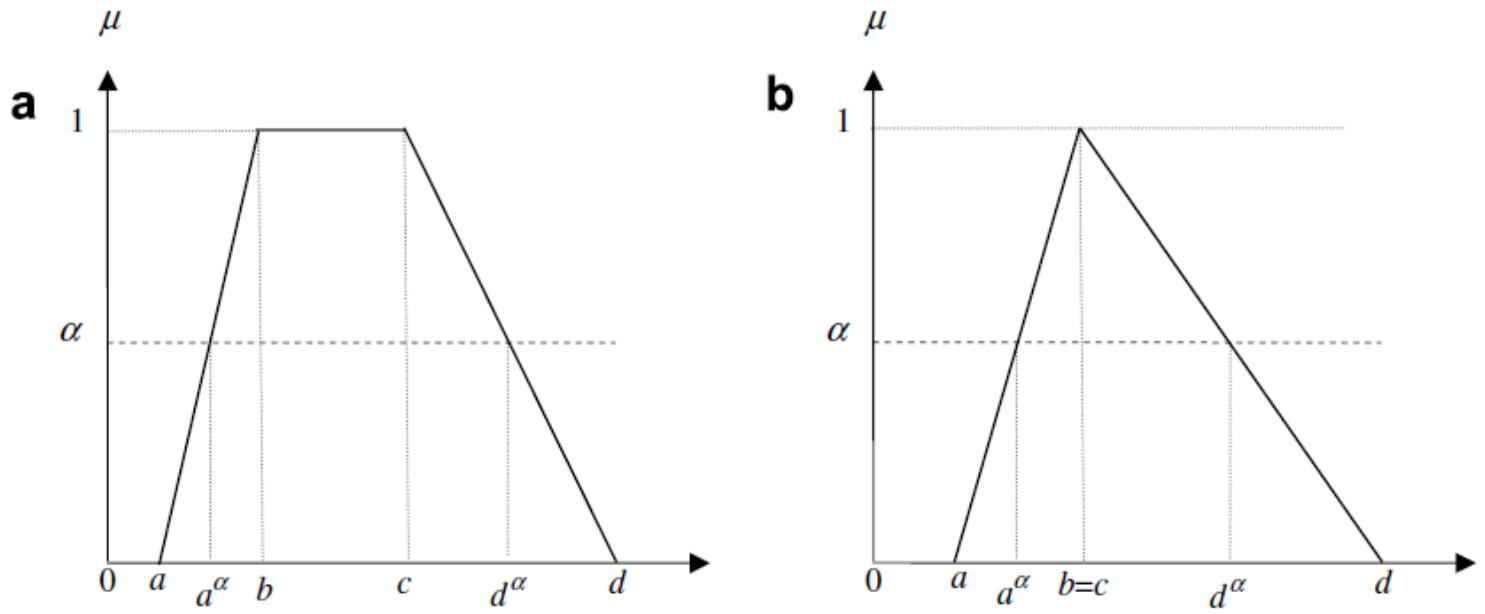


Fig. 1. Representation of a sample by trapezoidal and/or triangular fuzzy numbers: (a) Trapezoidal (a,b,c,d) and (b) triangular (a,b,b,d).

a,b,c,d為品質特徵， α-cuts為模糊係數

$$a^\alpha = a + \alpha(b-a)$$

$$d^\alpha = d - \alpha(d-c)$$

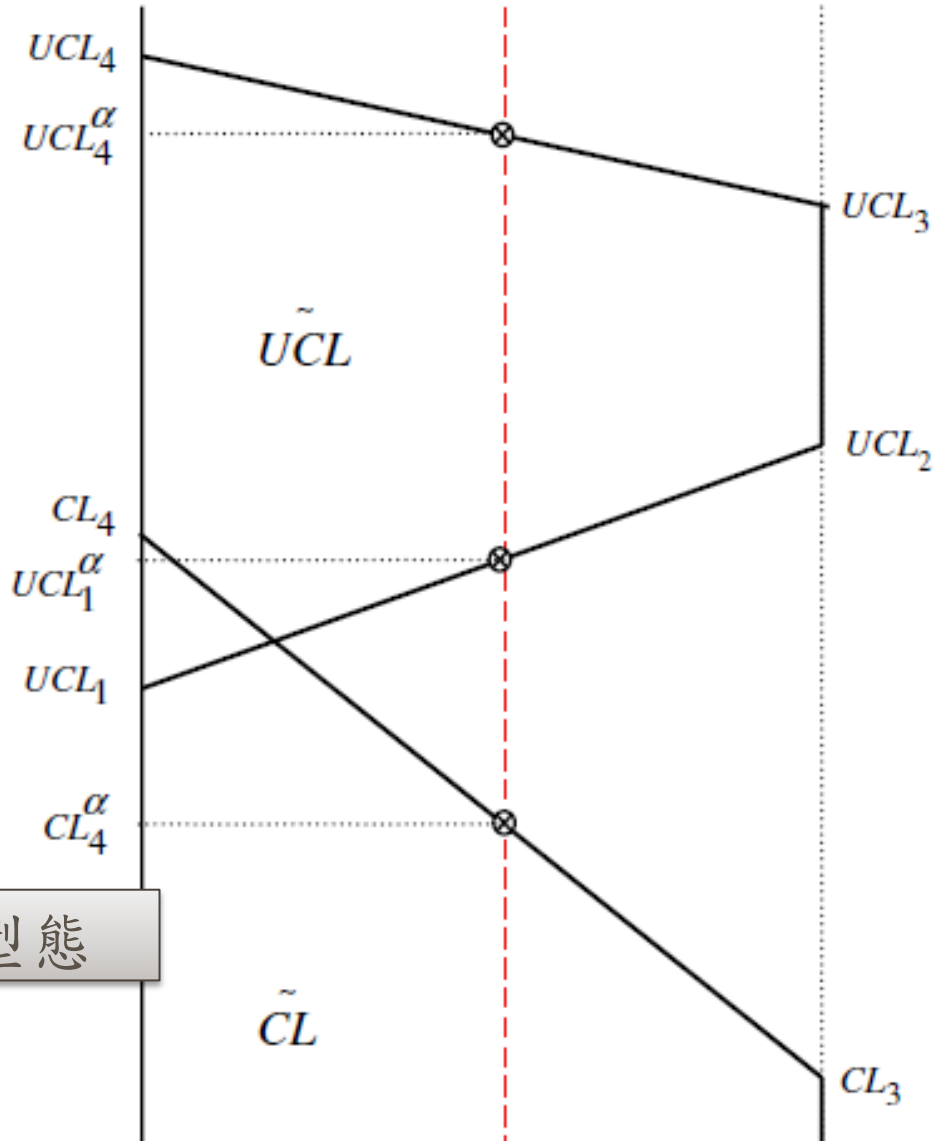
- 計算出α-cut fuzzy上下管制界線

$$\widetilde{\mathrm{CL}}^{\alpha} = (\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha}) = (\mathrm{CL}_1^{\alpha}, \mathrm{CL}_2, \mathrm{CL}_3, \mathrm{CL}_4^{\alpha}) \tag{10}$$

$$\widetilde{\mathrm{LCL}}^{\alpha} = \widetilde{\mathrm{CL}}^{\alpha} - 3\sqrt{\widetilde{\mathrm{CL}}^{\alpha}} = (\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha}) - 3\sqrt{(\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha})} = \left(\bar{a}^{\alpha} - 3\sqrt{\bar{d}^{\alpha}}, \bar{b} - 3\sqrt{\bar{c}}, \bar{c} - 3\sqrt{\bar{b}}, \bar{d}^{\alpha} - 3\sqrt{\bar{a}^{\alpha}}\right)$$
$$= (\mathrm{LCL}_1^{\alpha}, \mathrm{LCL}_2, \mathrm{LCL}_3, \mathrm{LCL}_4^{\alpha}) \tag{11}$$

$$\widetilde{\mathrm{UCL}}^{\alpha} = \widetilde{\mathrm{CL}}^{\alpha} + 3\sqrt{\widetilde{\mathrm{CL}}^{\alpha}} = (\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha}) + 3\sqrt{(\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha})} = \left(\bar{a}^{\alpha} + 3\sqrt{\bar{a}^{\alpha}}, \bar{b} + 3\sqrt{\bar{b}}, \bar{c} + 3\sqrt{\bar{c}}, \bar{d}^{\alpha} + 3\sqrt{\bar{d}^{\alpha}}\right)$$
$$= (\mathrm{UCL}_1^{\alpha}, \mathrm{UCL}_2, \mathrm{UCL}_3, \mathrm{UCL}_4^{\alpha}) \tag{12}$$

圖2



當α=1時接近傳統管制圖型態

2.1. Fuzzy control charts based on fuzzy mode transformation

- 模糊模式轉換
- $S_{mod,j}$其CL,LCL,UCL如下

$$S_{\mathrm{mod},j} = [b_j, c_j] \tag{14}$$

$$\mathrm{CL}_{\mathrm{mod}} = f_{\mathrm{mod}}(\widetilde{\mathrm{CL}}) = [\mathrm{CL}_2, \mathrm{CL}_3] \tag{15}$$

$$\mathrm{LCL}_{\mathrm{mod}} = \mathrm{CL}_{\mathrm{mod}} - 3\sqrt{\mathrm{CL}_{\mathrm{mod}}} = \left[(\mathrm{CL}_2 - 3\sqrt{\mathrm{CL}_2}), (\mathrm{CL}_3 - 3\sqrt{\mathrm{CL}_3})\right] = [\mathrm{LCL}_2, \mathrm{LCL}_3] \tag{16}$$

$$\mathrm{UCL}_{\mathrm{mod}} = \mathrm{CL}_{\mathrm{mod}} + 3\sqrt{\mathrm{CL}_{\mathrm{mod}}} = \left[(\mathrm{CL}_2 + 3\sqrt{\mathrm{CL}_2}), (\mathrm{CL}_3 + 3\sqrt{\mathrm{CL}_3})\right] = [\mathrm{UCL}_2, \mathrm{UCL}_3] \tag{17}$$

圖3

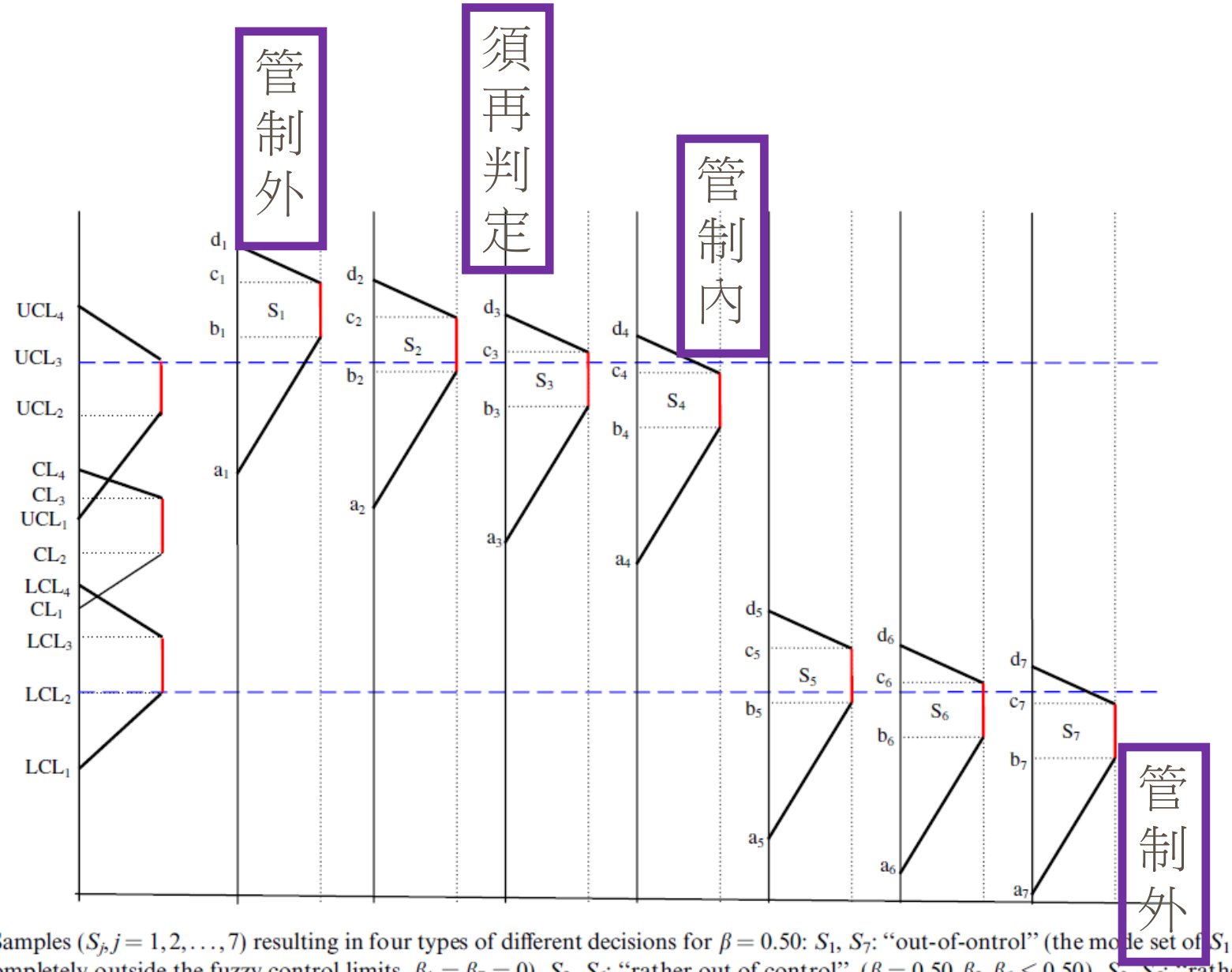


Fig. 3. Samples ($S_j, j = 1, 2, \ldots, 7$) resulting in four types of different decisions for $\beta = 0.50$: S_1, S_7: "out-of-ontrol" (the mode set of S_1 and S_7 are completely outside the fuzzy control limits, $\beta_1 = \beta_7 = 0$), S_2, S_6: "rather out of control", ($\beta = 0.50, \beta_2, \beta_6 \leqslant 0.50$), S_3, S_5: "rather in control" ($\beta = 0.50, \beta_3, \beta_5 \geqslant 0.50$), S_4: "in control" (mode set of S_4 is completely inside the fuzzy control limits, $\beta_4 = 1$).

- β越接近1，表示越明顯判斷，當β=1時，結果將會在管制內

$$\beta_j = \begin{cases} 0, & \text{for } b_j \geqslant \mathrm{UCL}_3 \\ \frac{\mathrm{UCL}_3 - b_j}{c_j - b_j}, & \text{for } (\mathrm{LCL}_2 \leqslant b_j \leqslant \mathrm{UCL}_3) \wedge (c_j \geqslant \mathrm{UCL}_3) \\ 1, & \text{for } (b_j \geqslant \mathrm{LCL}_2) \wedge (c_j \leqslant \mathrm{UCL}_3) \\ \frac{\mathrm{LCL}_2 - b_j}{c_j - b_j}, & \text{for } (b_j \leqslant \mathrm{LCL}_2) \wedge (\mathrm{LCL}_2 \leqslant c_j \leqslant \mathrm{UCL}_3) \\ 0, & \text{for } c_j \leqslant \mathrm{LCL}_2 \end{cases} \tag{18}$$

- 判定規則

$$\text{Process control} = \begin{cases} \text{in-control}, & \text{for } \beta = 1 \ (b_j \geqslant \mathrm{LCL}_2 \wedge c_j \leqslant \mathrm{UCL}_3) \\ \text{out-of-control}, & \text{for } \beta = 0 \ (b_j \geqslant \mathrm{UCL}_3 \vee c_j \leqslant \mathrm{LCL}_2) \\ \left.\begin{array}{l} \text{rather in-control}, \quad \text{for } \beta_j \geqslant \beta \\ \text{rather out-of-control}, \quad \text{for } \beta_j < \beta \end{array}\right\} & \text{otherwise} \end{cases} \tag{19}$$

2.2. Fuzzy control charts based on α-level fuzzy midrange transformation

- α-level模糊平均(midrange)轉換

$$f_{\mathrm{mr}}^{\alpha}=\frac{1}{2}(a^{\alpha}+d^{\alpha}) \tag{20}$$

- 其上下管制界線

$$\mathrm{CL}_{\mathrm{mr}}^{\alpha}=f_{\mathrm{mr}}^{\alpha}(\widetilde{\mathrm{CL}})=\frac{\mathrm{CL}_1^{\alpha}+\mathrm{CL}_4^{\alpha}}{2}=\frac{\mathrm{CL}_1+\mathrm{CL}_4+\alpha[(\mathrm{CL}_2-\mathrm{CL}_1)-(\mathrm{CL}_3-\mathrm{CL}_4)]}{2} \tag{22}$$

$$\mathrm{LCL}_{\mathrm{mr}}^{\alpha}=\mathrm{CL}_{\mathrm{mr}}^{\alpha}-3\sqrt{\mathrm{CL}_{\mathrm{mr}}^{\alpha}} \tag{23}$$

$$\mathrm{UCL}_{\mathrm{mr}}^{\alpha}=\mathrm{CL}_{\mathrm{mr}}^{\alpha}+3\sqrt{\mathrm{CL}_{\mathrm{mr}}^{\alpha}} \tag{24}$$

圖4

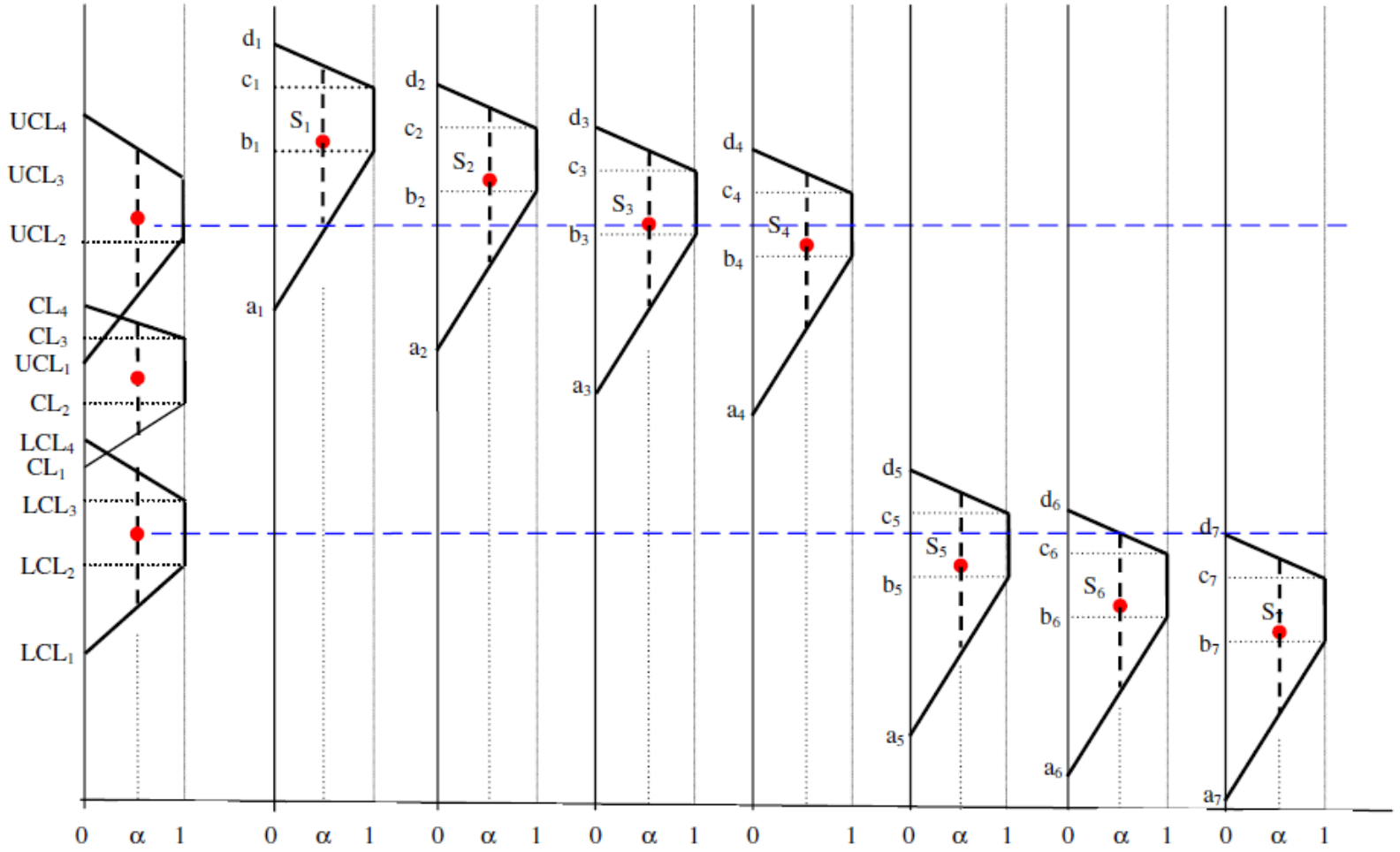


Fig. 4. Samples ($S_j, j = 1, 2, \ldots, 7$) resulting in two types of different decisions for $\alpha = 0.50$: S_1, S_2, S_5, S_6, S_7: "out-of-control" (α-level fuzzy midrange (●) of S_1, S_2, S_5, S_6, and S_7 outside the control limits), S_3, S_4: "in control" (α-level midrange of S_3 and S_4 are inside the control limits).

- 判定規則

$$\text{Process control} = \begin{cases} \text{in-control}, & \text{for } \text{LCL}_{\text{mr}}^{\alpha} \leqslant S_{\text{mr},j}^{\alpha} \leqslant \text{UCL}_{\text{mr}}^{\alpha} \\ \text{out-of-control}, & \text{otherwise} \end{cases} \tag{25}$$

2.3. Fuzzy control charts based on α-level fuzzy median transformation

- 模糊中位數轉換

$$S^{\alpha}_{\text{med},j} = \frac{1}{4}(a^{\alpha}_j + b_j + c_j + d^{\alpha}_j)$$

A1=A2

圖5

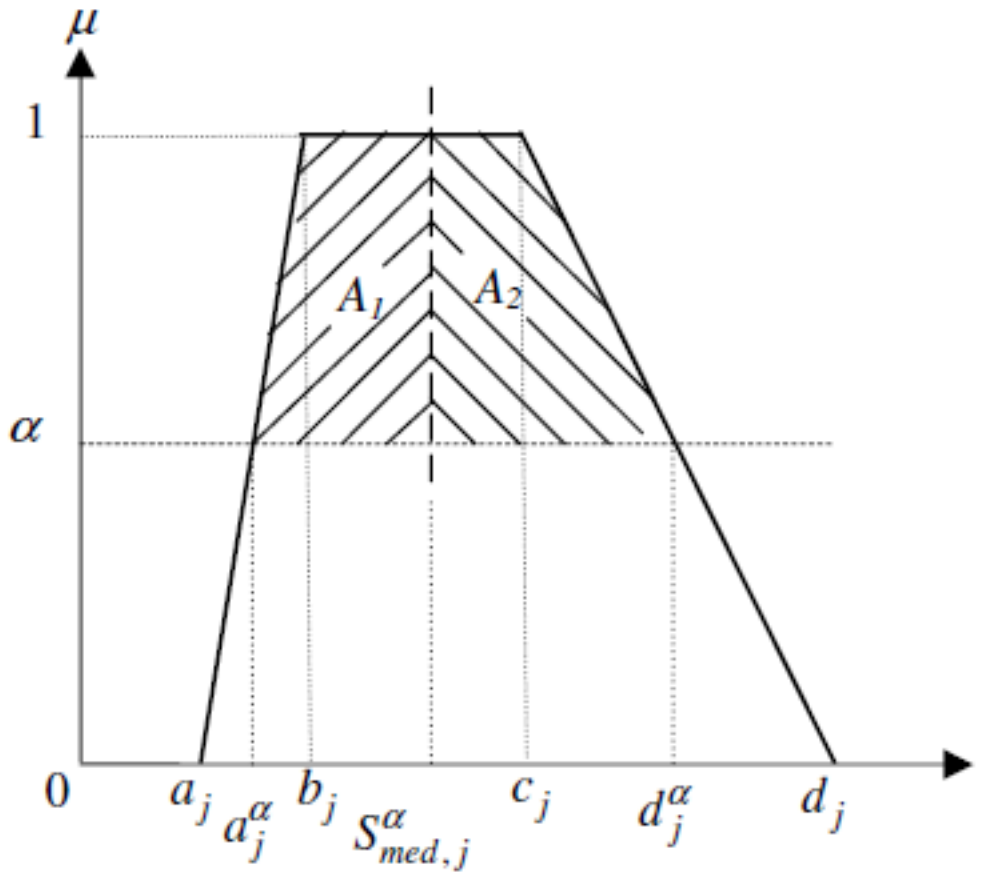


Fig. 5. Illustration of fuzzy median ($A_1 = A_2$).

- 上下管制界線

$$\mathrm{CL}_{\mathrm{med}}^{\alpha} = f_{\mathrm{med}}^{\alpha}(\widetilde{\mathrm{CL}}) = \frac{1}{4}(\mathrm{CL}_1^{\alpha} + \mathrm{CL}_2 + \mathrm{CL}_3 + \mathrm{CL}_4^{\alpha}) \tag{27}$$

$$\mathrm{LCL}_{\mathrm{med}}^{\alpha} = \mathrm{CL}_{\mathrm{med}}^{\alpha} - 3\sqrt{\mathrm{CL}_{\mathrm{med}}^{\alpha}} \tag{28}$$

$$\mathrm{UCL}_{\mathrm{med}}^{\alpha} = \mathrm{CL}_{\mathrm{med}}^{\alpha} + 3\sqrt{\mathrm{CL}_{\mathrm{med}}^{\alpha}} \tag{29}$$

圖6

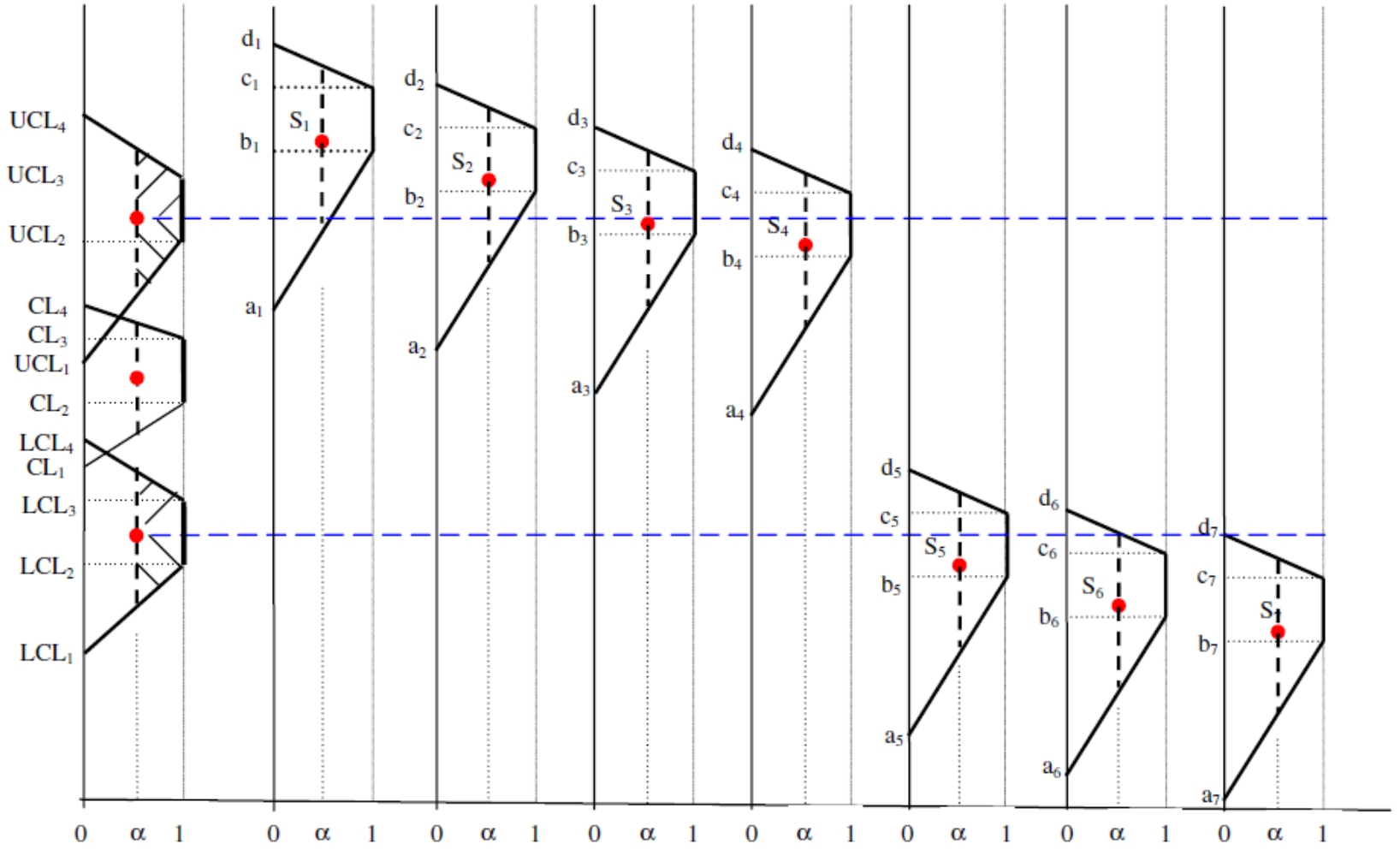


Fig. 6. Samples ($S_j, j = 1, 2, \ldots, 7$) resulting in two types of different decisions for $\alpha = 0.50$: S_1, S_2, S_5, S_6, S_7: "out-of-control" (α-level fuzzy median (●) of S_1, S_2, S_5, S_6, and S_7 outside the control limits), S_3, S_4: "in control" (the α-level fuzzy medians of S_3 and S_4 are inside the control limits).

- 判定規則

$$\text{Process control} = \begin{cases} \text{in-control}, & \text{for } \text{LCL}_{\text{med}}^{\alpha} \leqslant S_{\text{med},j}^{\alpha} \leqslant \text{UCL}_{\text{med}}^{\alpha} \\ \text{out-of-control}, & \text{otherwise} \end{cases} \tag{30}$$

3. A new approach to fuzzy control charts: direct fuzzy approach

- 為防止樣本因為轉換而失去訊息，因此提出直接模糊方法 (Direct fuzzy approach ; DFA)
- 使用上下管制界線如下

$$\widetilde{\mathrm{CL}}^{\alpha} = (\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha}) = (\mathrm{CL}_1^{\alpha}, \mathrm{CL}_2, \mathrm{CL}_3, \mathrm{CL}_4^{\alpha}) \tag{10}$$

$$\widetilde{\mathrm{LCL}}^{\alpha} = \widetilde{\mathrm{CL}}^{\alpha} - 3\sqrt{\widetilde{\mathrm{CL}}^{\alpha}} = (\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha}) - 3\sqrt{(\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha})} = \left(\bar{a}^{\alpha} - 3\sqrt{\bar{d}^{\alpha}}, \bar{b} - 3\sqrt{\bar{c}}, \bar{c} - 3\sqrt{\bar{b}}, \bar{d}^{\alpha} - 3\sqrt{\bar{a}^{\alpha}}\right)$$
$$= (\mathrm{LCL}_1^{\alpha}, \mathrm{LCL}_2, \mathrm{LCL}_3, \mathrm{LCL}_4^{\alpha}) \tag{11}$$

$$\widetilde{\mathrm{UCL}}^{\alpha} = \widetilde{\mathrm{CL}}^{\alpha} + 3\sqrt{\widetilde{\mathrm{CL}}^{\alpha}} = (\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha}) + 3\sqrt{(\bar{a}^{\alpha}, \bar{b}, \bar{c}, \bar{d}^{\alpha})} = \left(\bar{a}^{\alpha} + 3\sqrt{\bar{a}^{\alpha}}, \bar{b} + 3\sqrt{\bar{b}}, \bar{c} + 3\sqrt{\bar{c}}, \bar{d}^{\alpha} + 3\sqrt{\bar{d}^{\alpha}}\right)$$
$$= (\mathrm{UCL}_1^{\alpha}, \mathrm{UCL}_2, \mathrm{UCL}_3, \mathrm{UCL}_4^{\alpha}) \tag{12}$$

圖7

利用面積來做判定，方法與β相同

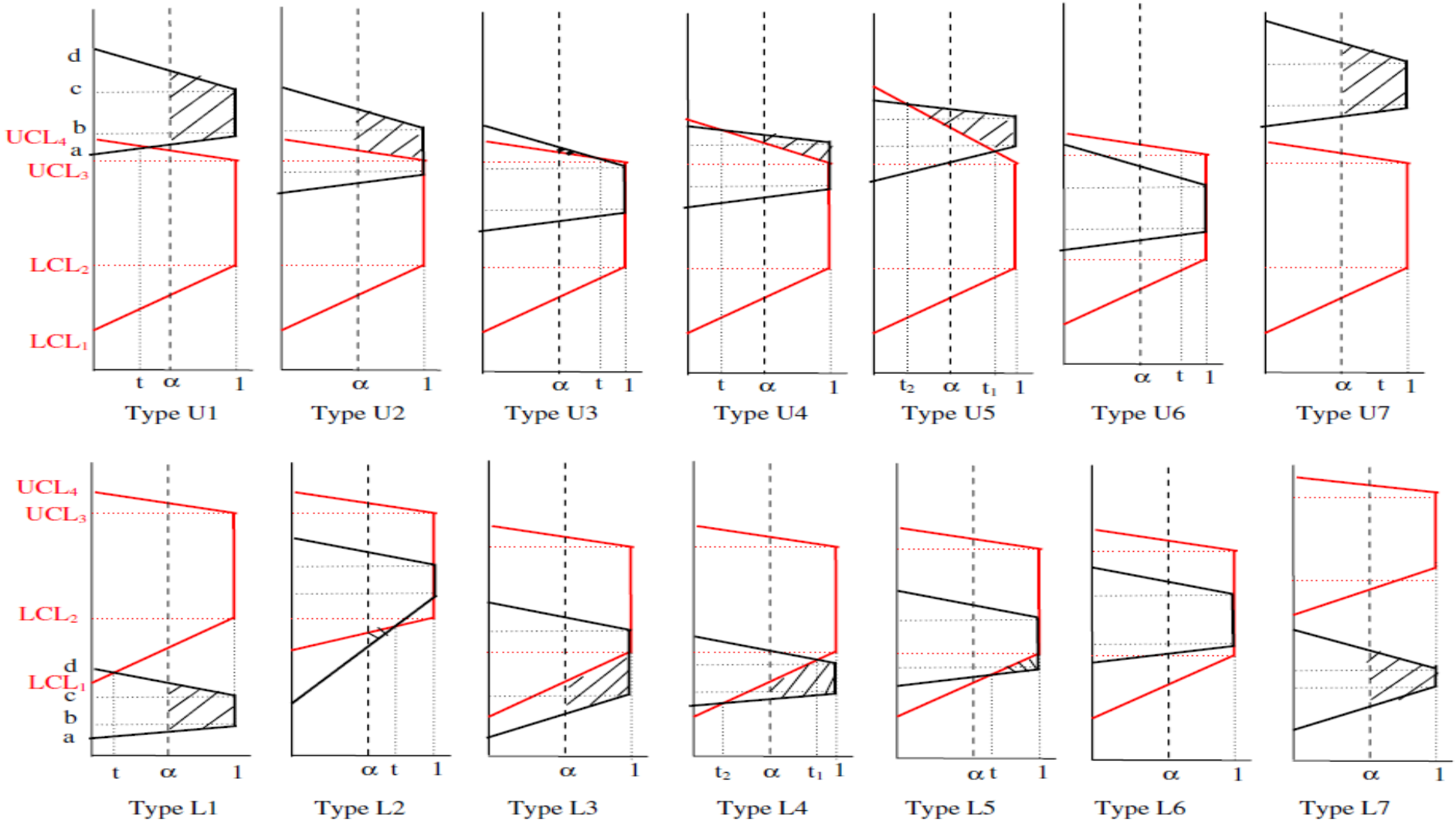
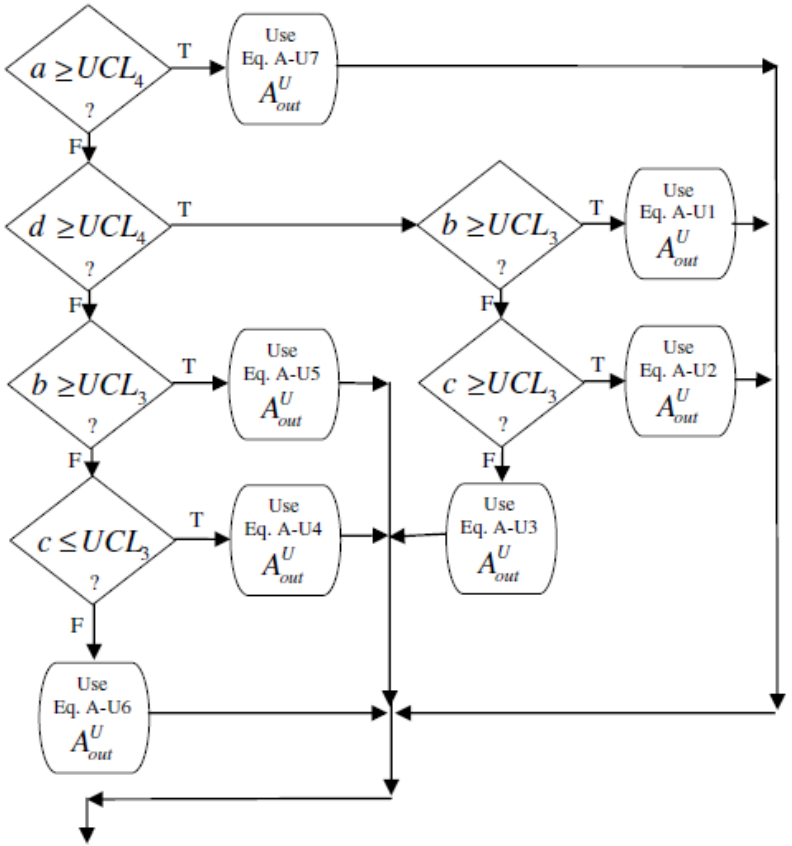


Fig. 7. Illustration of the possible areas outside the fuzzy control limits at α-level cut.

- 其計算流程如圖8


$a \geq UCL_4$?
T
Use Eq. A-U7 A_{out}^U
F
$d \geq UCL_4$?
T
$b \geq UCL_3$?
T
Use Eq. A-U1 A_{out}^U
F
$b \geq UCL_3$?
T
Use Eq. A-U5 A_{out}^U
$c \geq UCL_3$?
T
Use Eq. A-U2 A_{out}^U
F
F
$c \leq UCL_3$?
T
Use Eq. A-U4 A_{out}^U
Use Eq. A-U3 A_{out}^U
F
Use Eq. A-U6 A_{out}^U

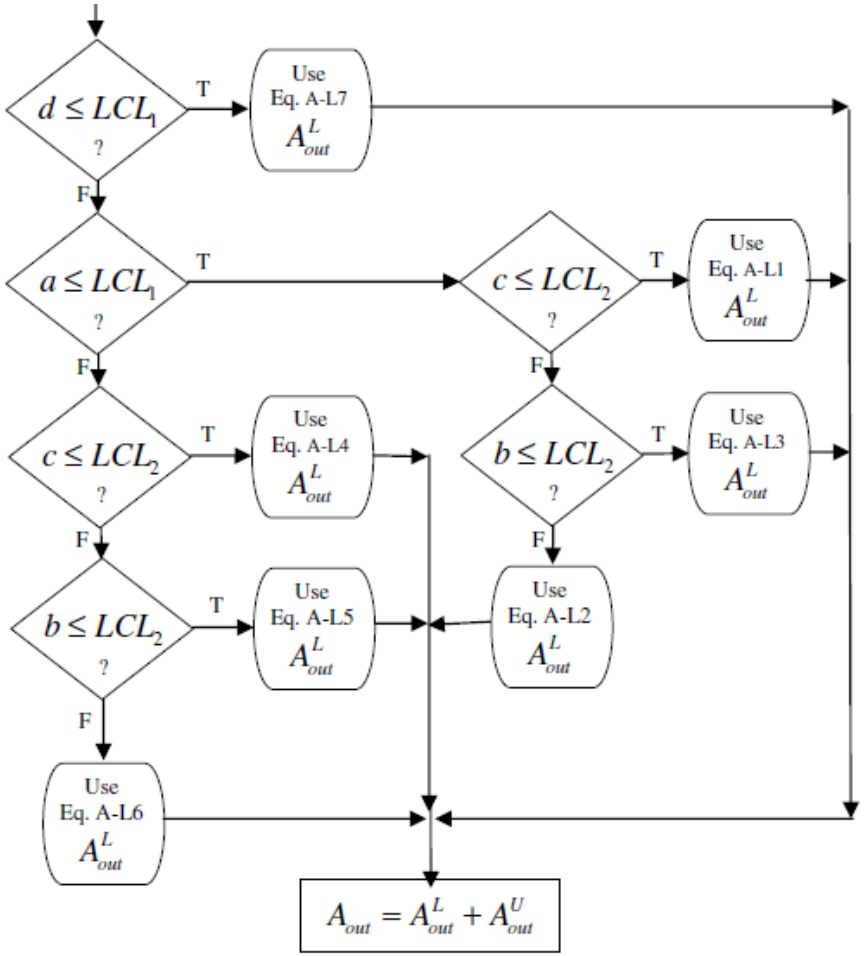


Fig. 8. Flowchart to compute the area of a fuzzy sample (a, b, c, d) falling outside the fuzzy control limits. (See appendix for the equations)

- The sample area above the upper control limits，$A_{\text{out}}^{\text{U}}$
- sample area falling below the lower control limits，$A_{\text{out}}^{\text{L}}$
- the total area outside the fuzzy control limits ，

$$A_{out} = A_{out}^{L} + A_{out}^{U}$$

4. A numerical example for number of nonconformities

- 從一家玩具公司生產的大型玩具每四小時來管制
- 所收集的數據來自 30個分組的語言，如表2

Table 2
Number of nonconformities for 30 subgroups

Sample no.	Approximately	Between
1	30	
2		20–30
3		5–12
4	6	
5	38	
6		20–24
7		4–8
8		36–44
9		11–15
10		10–13
11	6	
12	32	
13	13	
14		50–52
15		38–41

- 語言形式的表2代表的模糊值見表 3

Table 3
Fuzzy number (a,b,c,d) representation of 30 subgroups

No.	a	b	c	d
1	25	30	30	35
2	15	20	30	35
3	4	5	12	15
4	3	6	6	8
5	32	38	38	45
6	16	20	24	28
7	3	4	8	12
8	27	36	44	50
9	9	11	15	20
10	7	10	13	15
11	3	6	6	10
12	27	32	32	37
13	11	13	13	15
14	39	50	52	55
15	28	38	41	45

- The fuzzy modes, α-level fuzzy midranges, and α-level fuzzy medians of the fuzzy control limits 總結在表4

Table 4
Control limits and their representative values based on fuzzy mode, fuzzy midrange, and fuzzy median

	Fuzzy number				Fuzzy transformation method		
	a	*b*	*c*	*d*	Mode	Midrange ($\alpha = 0.60$)	Median ($\alpha = 0.60$)
CL	18, 13	22, 67	26, 93	32, 07	[22.67,26.93]	24.95	24.88
LCL	1, 15	7, 10	12, 65	19, 29	[7.10,12.65]	10.05	9.96
UCL	30, 91	36, 95	42, 5	49, 05	[36.95,42.50]	38.95	39.79

- fuzzy mode, α-level fuzzy midrange, and α-level fuzzy median決策結果如表5

Table 5
Decisions based on fuzzy mode, fuzzy midrange, and fuzzy median ($\alpha = 0.60$, $\beta = 0.70$)

S_j	$f_{\mathrm{mod},j}$		β_j	$f_{\mathrm{mod},j}$ decision	$f_{\mathrm{mr},j}^{\alpha=0.60}$	$f_{\mathrm{mr},j}^{\alpha=0.60}$ decision	$f_{\mathrm{med},j}^{\alpha=0.60}$	$f_{\mathrm{med},j}^{\alpha=0.60}$ decision
1	30	30	100.00	In control	30.00	In control	30.00	In control
2	20	30	100.00	In control	25.00	In control	25.00	In control
3	5	12	70.04	Rather in control	8.90	Out of control	8.70	Out of control
4	6	6	0.00	Out of control	5.80	Out of control	5.90	Out of control
5	38	38	100.00	In control	38.20	In control	38.10	In control
6	20	24	100.00	In control	22.00	In control	22.00	In control
7	4	8	22.56	Rather out of control	6.60	Out of control	6.30	Out of control
8	36	44	81.28	Rather in control	39.40	In control	39.70	In control
9	11	15	100.00	In control	13.60	In control	13.30	In control

- 各方法決策結果如表8-左半邊

Table 8
Comparison of alternative approaches: Fuzzy mode, fuzzy midrange,

j	$f_{\mathrm{mod},j}$ decision	$f_{\mathrm{mr},j}^{\alpha=0.60}$ decision
1	In control	In control
2	In control	In control
3	Rather in control	Out of control
4	Out of control	Out of control
5	In control	In control
6	In control	In control
7	Rather out of control	Out of control
8	Rather in control	In control
9	In control	In control
10	In control	In control
11	Out of control	Out of control
12	In control	In control
13	In control	In control
14	Out of control	Out of control
15	In control	In control

- 各方法決策結果如表8-右半邊

fuzzy median, and DFA ($\alpha = 0.60$ and $\beta = 0.70$)

$f_{\text{med},j}^{\alpha=0.60}$ decision	DFA ($\alpha = 0.60$) decision
In control	In control
In control	In control
Out of control	Rather in control
Out of control	Rather out of control
In control	In control
In control	In control
Out of control	Rather out of control
In control	Rather in control
In control	In control
In control	In control
Out of control	Rather in control
In control	In control
In control	In control
Out of control	Out of control
In control	In control

- 例如，樣本3在DFA(85.81%)判定為”Rather in control”，但其他的都判定為”out-of-control”

Table 7
Decisions based on direct fuzzy approach ($\alpha = 0.60$, $\beta = 0.70$)

S_j	a^α	b	c	d^α	Area out	Sample's area	β_j	DFA decision
1	28	30	30	32	0.00	0.80	100.00	In control
2	18	20	30	32	0.00	4.80	100.00	In control
3	4.6	5	12	13.2	0.44	3.12	85.81	Rather in control

- 可定義出製程管制規則

$$\text{Process control} = \begin{cases} \text{in-control}, & 0.85 \leqslant \beta_j \leqslant 1 \\ \text{rather in control}, & 0.60 \leqslant \beta_j < 0.85 \\ \text{rather out of control}, & 0.10 \leqslant \beta_j < 0.60 \\ \text{out-of-control}, & 0 \leqslant \beta_j < 0.10 \end{cases} \tag{32}$$

5. Conclusion

- DFA可防止樣本資訊遺失
- α-cut and an acceptable percentage (β)是能夠做管制圖判定之基準

THE END

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