A new monitoring design for uni-variate statistical quality control charts

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1. Introduction and literature review

- SPC用來監控製程品質，而發展至今已有Shewhart、CUSUM或EWMA
- CUSUM用來監控小偏移是較Shewhart佳

- 管制圖除了要能夠快速發現小偏移的發生外，假警報次數也不能太常發生
- 利用ARL即可觀察出管制內(ARL_0)或管制外(ARL_1)的長度

- In this research, a recursive equation is first defined to update a statistic (called belief) in each iteration of the data gathering process.
- Then, similar to the well-known *CUSUM* and *EWMA* methods, thresholds are derived for the updated values of the beliefs.
- When the updated belief is out of the derived threshold range, an out-of-control signal is issued.

2. Belief and the approach to its improvement

- n=1即觀察值只有一個
- 在第k個 iteration ，令$O_k=(x_1,x_2,\ldots,x_k)$
- 觀測值x_k
- $B(x_k,O_{k-1})$ define the belief in the process to be in an in-control state.

- In this iteration, our aim is to improve this belief based on the observation vector O_{k-1} and the new observation x_k

- 假設品質特性服從常態，其中mean μ_0 and variance σ^2
- $B(O_{k-1})=B(x_{k-1},O_{k-2})$ be the prior belief in an in-control state, in order to update the posterior belief $B(x_k,O_{k-1})$

$$B(x_k, O_{k-1}) = B(O_k) = \frac{B(O_{k-1})e^{\frac{x_k-\mu_0}{\sigma_0}}}{B(O_{k-1})e^{\frac{x_k-\mu_0}{\sigma_0}} + (1-B(O_{k-1}))} \tag{1}$$

- Then, by defining the statistic

$$Z_k = \frac{B(x_k, O_{k-1})}{1 - B(x_k, O_{k-1})} = \frac{B(O_k)}{1 - B(O_k)} \quad (2)$$

- the recursive equation(遞歸型方程式) will be

$$Z_k = e^{\frac{x_k - \mu_0}{\sigma_0}} \frac{B(O_{k-1})}{1 - B(O_{k-1})} = e^{\frac{x_k - \mu_0}{\sigma_0}} Z_{k-1} \quad (3)$$

- Hence,

$$Z_k = e^{\frac{x_k-\mu_0}{\sigma_0}} Z_{k-1} = e^{\frac{x_k-\mu_0}{\sigma_0}+\frac{x_{k-1}-\mu_0}{\sigma_0}} Z_{k-2} = \cdots = e^{\frac{\sum_{i=1}^{k} x_i - k\mu_0}{\sigma_0}} \tag{4}$$

- In other words,

$$Ln(Z_k) = \frac{\sum_{i=1}^{k} x_i - k\mu_0}{\sigma_0} = \sum_{i=1}^{k}\left(\frac{x_i-\mu_0}{\sigma_0}\right) \approx N(0,k) \tag{5}$$

- 從最初的Z_0 and $B(O_0)$，當k=1時，公式(4)寫為

$$Z_1 = e^{\frac{\sum_{i=1}^{1} x_i - \mu_0}{\sigma_0}} = e^{\frac{x_1 - \mu_0}{\sigma_0}}$$

- 而公式(3)寫為 $Z_1 = e^{\frac{x_1 - \mu_0}{\sigma_0}} Z_0$
- 因此Z_0=1 ，則 $B(O_0)$=0.5

- 定義上下管制界線 (*UCL* and *LCL*) for *Ln(Z_k)*

$$UCL_{Ln(Z_k)} = c\sqrt{k} \text{ and } LCL_{Ln(Z_k)} = -c\sqrt{k} \tag{6}$$

- 其中c 是一個 multiple of the standard deviation
- 而*Ln(Z_k)*之信賴水準為

$$P(-c\sqrt{k} \leqslant Ln(z_k) \leqslant c\sqrt{k}) = 1 - \alpha \tag{7}$$

- 帶入Z_k得

$$P\left[-c\sqrt{k} \leqslant Ln\left(\frac{B(x_k, O_{k-1})}{1 - B(x_k, O_{k-1})}\right) \leqslant c\sqrt{k}\right] = 1 - \alpha$$

(8)

- 整理過後

$$P\left[\frac{e^{-c\sqrt{k}}}{(e^{-c\sqrt{k}} + 1)} \leqslant B(x_k, O_{k-1}) \leqslant \frac{e^{c\sqrt{k}}}{(1 + e^{c\sqrt{k}})}\right] = 1 - \alpha$$

(11)

- 由於在初始階段所產生的假警報率可能很高，因此加入*l* 來控制假警報率

$$UCL_{B(x_k,O_{k-1})} = \frac{e^{-c\sqrt{k+l}}}{(1+e^{-c\sqrt{k+l}})} \text{ and } LCL_{B(x_k,O_{k-1})} = \frac{e^{c\sqrt{k+l}}}{(1+e^{c\sqrt{k+l}})} \quad (12)$$

- 在此*c*與*l*是必須做合理的設定，才能有效的將此建議方法之性能表現出來

3. Simulation experiments

- 模擬實驗進行兩種類別
  1. 一是獨立標準常態
  2. 二是自相關 *AR*(1) observations

3.1. Independent standard normal process

假設數據為獨立且常態時， pairs of independent uniform random variates

$$((R_i, R_{i+1});\ i = 2k - 1;\ k = 1, 2, 3, \ldots)$$

are first generated and then

$$x_k = \sqrt{-2Ln(R_i)}\cos(2\pi R_{i+1})$$

用來生成在標準常態下之第*k*個iteration觀測值

- 接著使用Eq. (1) the belief ($B(O_k)$) is updated in that iteration
- 當超出UCL&LCL即發生失控點

- 在此模擬實驗中EWMA與CUSUM參考值(reference value)為1
- 利用模擬，比較了*GEWMA* control charts、optimal *EWMA* 、Shewhart *EWMA* 、*GLR* 、and *CUSUM*

SD= standard deviations of the run lengths

- 其結果如下

Table 1. The results of ARL_0 and ARL_1 study for IID $N(0,1)$ observations.

Shifts	Proposed method	SD	Optimal EWMA	SD	Shewhart EWMA	SD	GEWMA	SD	GLR	SD	CUSUM	SD
0.00	548.00	21.370	437.00	4.34	430.00	4.28	438.00	4.24	439.00	4.35	434.00	4.36
0.10	63.50	0.880	297.00	4.30	294.00	2.85	304.00	2.75	295.00	2.67	326.00	3.23
0.25	23.01	0.230	110.00	1.02	109.00	1.02	105.00	0.79	108.00	0.80	132.00	1.23
0.50	8.55	0.070	32.40	0.25	32.40	0.25	34.90	0.23	36.20	0.23	37.20	0.30
0.75	4.66	0.030	15.70	0.10	15.70	0.10	17.40	0.10	18.10	0.11	16.70	0.11
1.00	3.15	0.020	9.95	0.05	9.92	0.05	10.70	0.06	11.10	0.06	10.30	0.05
1.25	2.51	0.010	7.24	0.03	7.19	0.03	7.36	0.04	7.58	0.04	7.34	0.03
1.50	1.93	0.015	5.37	0.02	5.67	0.02	5.41	0.03	5.59	0.03	5.70	0.02
2.00	1.40	0.009	4.03	0.01	3.91	0.01	3.41	0.02	3.54	0.02	3.98	0.01
3.00	1.04	0.007	2.63	0.01	2.29	0.01	1.85	0.01	1.91	0.01	2.55	0.01
Parameters	c=1.5 l=0		λ=0.128 L=2.82		λ=0.128 C=2.82 L=3.9		L=3.29		Z=3.45		H=4.94	

3.2. Auto-correlated AR(1) process

- 假設觀測值在不同階段所收集到品質特徵可作為一個AR(1)加上常態隨機誤差
- A process {y_k}is said to be *AR*(1) if it is generated by $y_k - \mu_0 = \varphi(y_{k-1} - \mu_0) + \varepsilon_k$ (13)
- 其中 *φ* is the autocorrelation coefficient ，範圍*φ*(-1,1)
- ε_k is a sequence of IID normal error term

- 在此程序中， the variance of the observations is $Var(y_k) = \frac{\sigma_\varepsilon^2}{1-\varphi^2}$
- 而殘差定義為

$$\varepsilon_k = y_k - \mu_0 - \varphi(y_{k-1} - \mu_0); \quad k = 1, 2, \ldots \tag{14}$$

- 從上述殘差已即IID隨機變數， 代入($B(y_k,O_{k-1})$)後得到如下式

$$B(y_k, O_{k-1}) = \frac{B(O_{k-1})e^{\frac{\varepsilon_k}{\sigma_\varepsilon}}}{B(O_{k-1})e^{\frac{\varepsilon_k}{\sigma_\varepsilon}} + (1 - B(O_{k-1}))} = \frac{B(O_{k-1})e^{\frac{y_k - \mu_0 - \varphi(y_{k-1} - \mu_0)}{\sigma_\varepsilon}}}{B(O_{k-1})e^{\frac{y_k - \mu_0 - \varphi(y_{k-1} - \mu_0)}{\sigma_\varepsilon}} + (1 - B(O_{k-1}))} \quad (15)$$

- 因此可得出100(1-α)% confidence interval

$$P\left[\frac{e^{-c\sqrt{k+l}}}{(e^{-c\sqrt{k+l}} + 1)} \leqslant B(y_k, O_{k-1}) \leqslant \frac{e^{c\sqrt{k+l}}}{(1 + e^{c\sqrt{k+l}})}\right] = 1 - \alpha \quad (16)$$

- 利用模擬，比較了residual-based *EWMA* chart [15], residual-based *CUSUM* chart [16] and one-sided *CUSCORE* chart [25] for selected auto-correlation coefficients of 0.1, 0.5 and 0.9

Table 2. The results of ARL_0 and ARL_1 study for $AR(1)$ with Φ=0.5.

Shifts	Proposed method	SD	Residual-based EWMA	SD	CUSCORE	SD	Residual-based CUSUM	SD
0.00	451.00	17.37	421.00	4.17	420.00	4.35	430.00	4.26
0.10	93.50	1.24	257.00	2.59	295.00	2.67	301.00	2.83
0.25	32.01	0.28	134.47	1.22	144.532	1.30	185.201	1.73
0.50	15.55	0.1	67.328	0.55	67.282	0.5	89.979	0.80
0.75	10.66	0.06	36.8727	0.30	36.677	0.26	48.2835	0.41
1.00	7.27	0.03	23.682	0.16	26.22	0.15	29.3704	0.23
1.25	5.51	0.02	17.4242	0.11	17.944	0.1	19.8704	0.14
1.50	4.93	0.015	13.1241	0.07	14.493	0.07	14.2967	0.09
2.00	3.60	0.008	4.93	0.04	3.9	0.04	8.98	0.05
3.00	2.4	0.005	3.32	0.02	1.8	0.02	4.05	0.02
Parameters	c=1.2 l=20		λ=0.1 L=2.51		L=3.5 k=0.25 r=0.5		H=4.25 k=0.5	

Table 3. The results of ARL_0 and ARL_1 study for $AR(1)$ with ϕ=0.9.

Shifts	Proposed method	SD	Residual-based EWMA	SD	CUSCORE	SD	Residual-based CUSUM	SD
0.00	451.00	17.37	418.00	4.25	443.00	4.05	426.00	4.19
0.10	394.63	11.24	386.00	3.97	393.00	3.5	391.00	3.83
0.25	291.24	6.77	330.35	3.34	285.02	3.22	358.11	3.48
0.50	178.13	3.34	268.02	2.69	201.48	2.55	301.27	3.01
0.75	127.09	1.90	211.28	2.12	146.79	2.09	256.89	2.49
1.00	91.84	1.22	177.96	1.69	115.40	1.70	217.42	2.09
1.25	72.14	0.85	144.45	1.31	88.62	1.35	183.75	1.79
1.50	60.18	0.63	116.64	1.11	72.97	1.10	160.25	1.52
2.00	38.1	0.41	85.93	0.72	82.9	0.47	118.98	1.13
3.00	28.2	0.23	51.32	0.39	55.8	0.26	63.05	0.63
					L=1.45			
Parameters	c=1.2		λ=0.1		k=0.05		H=4.25	
	l=20		L=2.51		r=0.1		k=0.5	

Table 4. The results of ARL_0 and ARL_1 study for $AR(1)$ with ϕ=0.1.

Shifts	Proposed method	SD	Residual-based EWMA	SD	CUSCORE	SD	Residual-based CUSUM	SD
0.00	451.00	17.37	425.00	4.37	445.00	4.05	428.00	4.17
0.10	103.63	1.54	189.4.00	1.87	232.00	2.22	391.00	3.83
0.25	40.04	0.36	69.01	0.62	100.30	0.40	101.25	0.97
0.50	17.68	0.11	27.51	0.21	33.63	0.14	35.33	0.30
0.75	10.87	0.05	15.83	0.08	16.84	0.08	17.22	0.12
1.00	8.29	0.04	10.57	0.05	10.54	0.05	10.44	0.06
1.25	6.30	0.03	7.70	0.03	7.64	0.03	7.49	0.04
1.50	5.25	0.02	6.43	0.03	5.93	0.03	5.74	0.03
2.00	4	0.01	4.39	0.01	4.15	0.01	3.98	0.01
3.00	2.67	0.01	2.84	0.01	2.67	0.01	2.54	0.01
Parameters	c=1.2 l=20		λ=0.1 L=2.51		L=4.2 k=0.45 r=0.9		H=4.25 k=0.5	

4. A case study

- process shifts to the mean μ_1=0.1
- 表5為20個觀測值
- 比較standard Shewhart, *CUSUM*, and *EWMA* charts

Table 5. The process data.

No.	Obs.	No.	Obs.
1	−0.4037	11	−0.9202
2	1.6336	12	0.4118
3	−0.2238	13	0.7242
4	0.7271	14	1.6376
5	−1.3756	15	1.8857
6	−0.7735	16	0.6630
7	1.8541	17	0.4673
8	0.5053	18	1.8906
9	−0.1224	19	−2.5656
10	0.5932	20	−0.5820

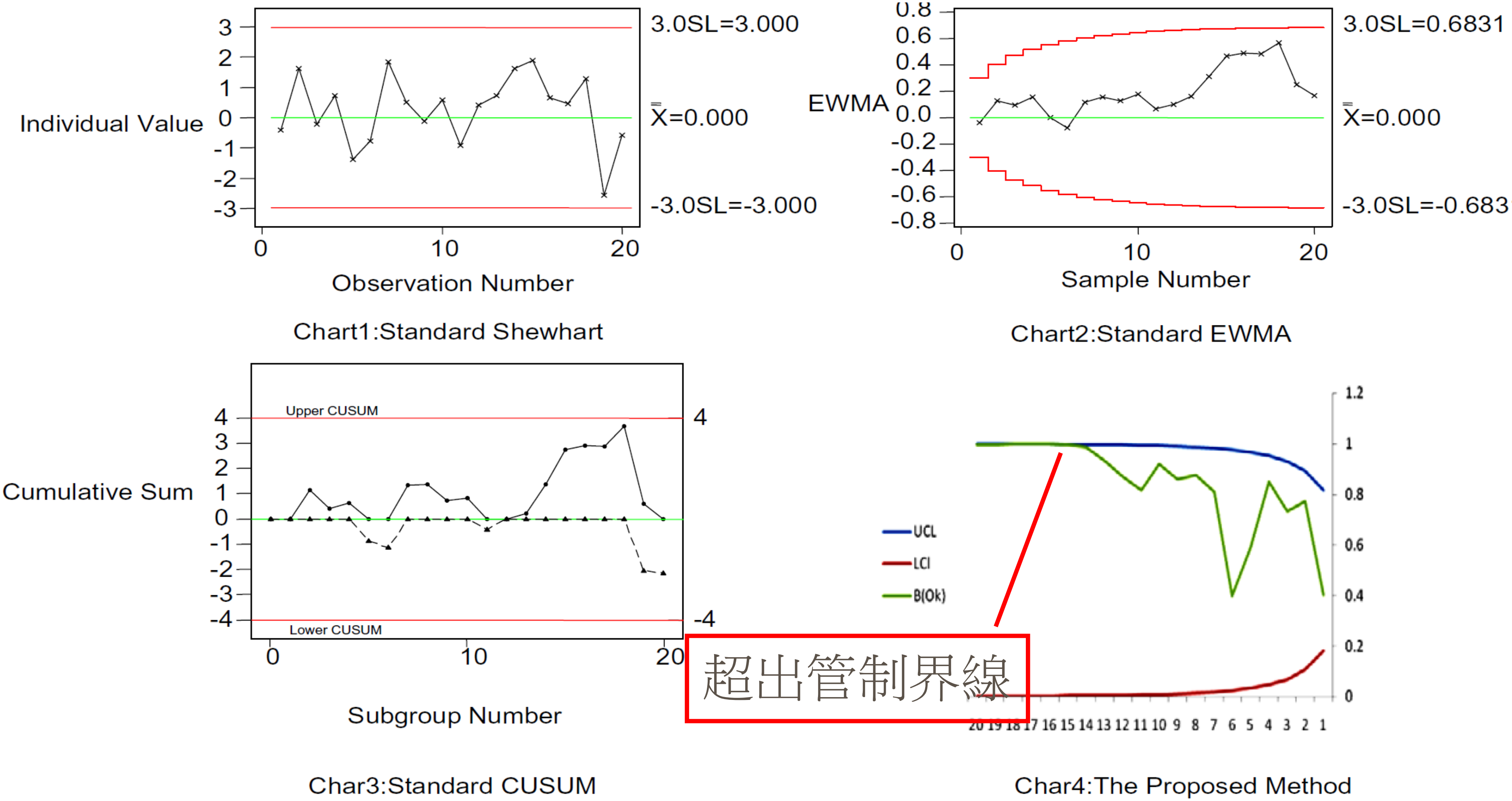

Fig. 1. The control charts of the case study.

- 沒有任何一個傳統管制圖可偵測出small shift of 0.1 in the process mean
- 這說明了建議的管制圖是更能夠檢測製程偏移且更實用

5. Conclusions and recommendation for future research

- 使用此方法能夠提升管制圖之性能，也可能降低型I型以及II誤差
- 未來研究可假設為無母數樣本

THE END

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