On the monitoring of multi-attributes high-quality production processes

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- 1.Introduction and literature review
- 2.Process monitoring based on simultaneous *CCC* and *C* charts
- 3.The proposed normalizing transformation
- 4.Multi-attribute control chart based on transformed data in $\chi 2$ chart
- 5.Numerical examples
- 6.Conclusion and recommendations for future research

1.Introduction and literature review

- 在高品質的製程裡，去找出不合格項和缺陷數是非常重要的。
- He et al. (2002)對高品質的製程提出以GPD (Generalized Poisson distribution)為基礎的不合格項的缺陷數管制圖。
- Patel (1973) proposed a Hotelling-type χ^2 chart to monitor observations from multivariate Binomial or multivariate Poisson distribution(for time independent and dependent samples).

1.Introduction and literature review

- Lu et al. (1998)提出一個多元計數的M-np管制圖，因為計數間是相關的，所以在減少type II errors方面比個別np管制圖好，然而在他們的研究論文未討論到M-np管制圖的ARL及統計分配函數。
- However, none of the proposed methods so far is able to monitor multi-attribute high quality processes(2007).

2.Process monitoring based on simultaneous CCC and C charts

- He et al. (2002)提出了一個製程和同時使用兩個單獨的控制圖,

Generalized Poisson Distribution (GPD) models

$$P_x(\theta,\lambda)=\frac{\theta(\theta+x\lambda)^{x-1}e^{-\theta-x\lambda}}{x!},\quad x=0,1,2\ldots,\ \lambda\geqslant 0 \qquad (1)$$

不合格品的機率(p)：$1-e^{-\theta}$，(θ增加，製程不良率便會增加)，因此，θ shows the defective rate of the process.

2.Process monitoring based on simultaneous *CCC* and *C* charts

- He .et al. (2002) apply two separate control charts to monitor parameters of a GPD.
- They use a geometric chart (*CCC* chart) to monitor parameter θ and apply a *C* chart to control λ. In the *CCC* chart,

$$UCL_{CCC} = \frac{\ln\left(\frac{\alpha_{CCC}}{2}\right)}{\ln(1-p)}, \quad CL_{CCC} = \frac{\ln(0.5)}{\ln(1-p)}, \quad \text{and}$$

$$LCL_{CCC} = \frac{\ln\left(1-\frac{\alpha_{CCC}}{2}\right)}{\ln(1-p)} \qquad (2)$$

α_{CCC} ： false alarm rate

2.Process monitoring based on simultaneous *CCC* and *C* charts

- In the *C* chart, we obtain the control limits by conditioning：

$$P(k \text{ nonconformities in a product} \mid \text{the product is defective}) = P(X = k \mid X > 0) =$$

$$\frac{\theta(\theta + k\lambda)^{k-1} e^{-\theta - k\lambda}}{k!(1 - e^{-\theta})}, \quad k = 1, 2, \ldots$$

This is called zero-truncated generalized Poisson distribution

(去零的一般波松分配)

$$E(X) = \theta(1-\lambda)^{-1}(1-e^{-\theta})^{-1}$$

$$\mathrm{Var}(X) = \left[\theta(1-\lambda)^{-3} + \theta^2(1-\lambda)^{-2}\right](1-e^{-\theta})^{-1} - \theta^2(1-\lambda)^{-2}(1-e^{-\theta})^{-2} \quad (3)$$

$$UCL_C = E(X) + 3\sqrt{\mathrm{Var}(X)}, \quad CL_C = E(X), \quad \text{and}$$

$$LCL_C = E(X) - 3\sqrt{\mathrm{Var}(X)} \quad (4)$$

3.The proposed normalizing transformation

- 如果資料是二項、波松或幾何分配，而我們假設它是常態，那會引起二個問題：

1.The fact that these distributions have skewnesses.
2.The discrete nature of these distributions.

3.The proposed normalizing transformation

- 在單一變數的計數管制圖裡，有兩個方法可以減少偏態：

1. adding correction values to the control limits based on the value of the skewness.
2. **applying normalizing transformation**

Since using normal data in statistical process control has many advantages.

然而大部份的研究者比較喜歡使用”常態轉換的技巧”在
處理偏態的問題。

3. The Proposed Normalizing Transformation

學者	年代	轉換方法
Box and Cox	1964	square root
Johnson and Kotz	1969	inverse 反函數法
Ryan	1989	arcsin 反正弦法
Ryan and Schwertman	1997	parabolic inverse 拋物線反置法
Quesenberry	1995	Q-transformation Q-轉置法
Xie et al.	2000	**double square root** transformation for Geometric distribution
Niaki and Abbasi	2007	rth root transformation

3.The proposed normalizing transformation

- In our first method of finding a proper transformation, using simulated data we employ a bisection method (Thisted 2000) to find the power of the root transformation for each attribute.
- The second proposed method of finding a proper transformation is similar to *Q*-transformation proposed by Quesenberry (1995). In this method, which we call the NORTA inverse transformation method.

3.The proposed normalizing transformation

- the vector **X** by a transformation of a k-dimensional standard multivariate normal (MVN) vector $[x_1, x_2, \ldots, x_k]^T$

$$\mathbf{X} = \begin{pmatrix} \Phi^{-1}(F_{x_1}(x_1)) \\ \Phi^{-1}(F_{x_2}(x_2)) \\ \vdots \\ \Phi^{-1}(F_{x_k}(x_k)) \end{pmatrix} \qquad (6)$$

$$E(x_i x_j) = E\left\{F_{x_i}^{-1}[\Phi(z_i)]F_{x_j}^{-1}[\Phi(z_j)]\right\}$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_{x_i}^{-1}[\Phi(z_i)]F_{x_j}^{-1}[\Phi(z_j)]\varphi_{\rho_z(i,j)}(z_i, z_j)\mathrm{d}z_i\,\mathrm{d}z_j \qquad (7)$$

3.The proposed normalizing transformation

Table 1 Covariance matrix and mean vector before and after transformation in example 1

	Before transformation	After root transformation $r_1 = 0.2868$ $r_2 = 0.7021$	After NORTA inverse transformation
$\mu_{\mathbf{X}}$	$[1000, 2]$	$[6.555, 1.508]$	$[0.0201, 0.3670]$
$\mathrm{Cov}(\mathbf{X})$	$\begin{pmatrix} 1065000 & 527.95 \\ 527.95 & 1.9639 \end{pmatrix}$	$\begin{pmatrix} 4.399 & 0.7146 \\ 0.7146 & 0.7336 \end{pmatrix}$	$\begin{pmatrix} 1.0168 & 0.3625 \\ 0.3625 & 0.8192 \end{pmatrix}$

Table 2 Covariance matrix and mean vector before and after transformation in example 2

	Before transformation	After root transformation $r_1 = 0.2698,\ r_2 = 0.7056$	After NORTA inverse transformation
$\mu_{\mathbf{X}}$	$[10000, 2]$	$[10.8561, 3.0532]$	$[0.0066, 0.2380]$
$\mathrm{Cov}(\mathbf{X})$	$\begin{pmatrix} 100260000 & -10787 \\ -10787 & 5.0786 \end{pmatrix}$	$\begin{pmatrix} 10.7162 & -1.8228 \\ -1.8228 & 1.0127 \end{pmatrix}$	$\begin{pmatrix} 1.0090 & -0.5427 \\ -0.5427 & 0.9531 \end{pmatrix}$

3.The proposed normalizing transformation

Table 3 A comparison of different transformation methods

		Index	Method: X^r transformation	$\sqrt[4]{X}, \sqrt{X}$	arcsin(X)	NORTA inverse
Example 1	X_1	Skewness	0.0103	−0.118	1.2089	0.0214
		Kurtosis	2.7947	2.8675	5.5866	3.0085
		P value(JB test)	0.0014	0	0	0.6743
	X_2	Skewness	−0.0154	−0.6747	1.4724	0.2337
		Kurtosis	2.7960	3.0662	1.8444	2.7753
		P value (JB test)	0.0013	0	0	0
Example 2	X_1	Skewness	−0.0393	−0.1018	1.1537	−0.0538
		Kurtosis	2.6917	2.7212	5.4018	2.9423
		P value (JB test)	0	0	0	0.4202
	X_2	Skewness	0.0252	−0.424	1.3015	0.0630
		Kurtosis	3.0348	3.8705	4.3454	2.9046
		P value (JB test)	0.4614	0	0	0.0054

First, the skewnesses and the kurtosises of the transformed data are always closer to 0 and 3.

Second, in all cases, the *p* values of the JB test in the proposed methods are at least better than those of the other transformation methods.

4.Multi-attribute control chart based on transformed data in χ2 chart

- χ^2 and T^2 control charts, 被廣泛的應用在多變數的品質管制環境，當然在我們轉換多元計數製程的多元常態分配也會表現優良.
- 雖然Patel’s method (1973)- T^2 control charts, 是一個不錯的方法在解決這類的問題上，但因為它忽略了一些限制，且使用近似常態的假設，所以當高品質的製程被監控時，是相當不精確的。

4.Multi-attribute control chart based on transformed data in $\chi 2$ chart

- To avoid this problem, we first eliminate the skewness of the marginal distributions, and then estimate the mean vector and the covariance matrix of the transformed attributes used in the well known χ^2 control chart.

5.Numerical examples

5.1 Example 3

generate 5,000 observations

Table 5 *ARL* values for different mean shifts in example 3

Shift→	(0,0)	$0.5(-\sigma_1,0)$	$0.5(0,\sigma_2)$	$(-0.5\sigma_1,\sigma_2)$	$0.5(\sigma_1,0)$	$0.5(0,\sigma_2)$	$0.5(\sigma_1,\sigma_2)$
Root transformation	21.23	17.52	8.61	6.98	12.05	11.57	10.08
NORTA inverse	20.67	16.62	8.13	6.83	10.86	10.24	9.94
CCC & C	22.7	26.24	15.11	14.20	14.67	16.45	11.10
Shift→	$(\sigma_1,0)$	$(0,\sigma_2)$	(σ_1,σ_2)	$1.5(\sigma_1,0)$	$1.5(0,\sigma_2)$	$1.5(\sigma_1,\sigma_2)$	$(2\sigma_1,0)$
Root transformation	6.49	5.40	5.30	4.41	3.00	3.28	3.39
NORTA inverse	6.46	5.24	5.01	4.08	3.04	3.22	3.26
CCC & C	9.57	8.105	5.57	5.45	4.43	3.40	4.2
Shift→	$(0,2\sigma_2)$	$(2\sigma_1,2\sigma_2)$	$(3\sigma_1,0)$	$(0,3\sigma_2)$	$(3\sigma_1,3\sigma_2)$	$0.5(-\sigma_1,\sigma_2)$	$(\sigma_1,-\sigma_2)$
Root transformation	2.02	2.26	2.36	1.31	1.45	7.03	2.37
NORTA inverse	1.92	2.27	2.40	1.29	1.45	6.69	2.52
CCC & C	2.99	2.42	2.89	1.73	1.59	13.62	8.31

Geometric probability distribution is p = 0.00015.Poisson distribution with mean three(λ=3). Since the ARL_0 values are appropriate, we compare the out-of-control average run length (ARL_1) values of the proposed methods for different shifts.

$$\hat{\Sigma} = \begin{pmatrix} 1 & 0.55 \\ 0.55 & 1 \end{pmatrix}.$$

5.Numerical examples

Table 6 *AIRL* values for different mean shifts in example 3

Shift→	$(0,0)$	$0.5(-\sigma_1,0)$	$0.5(0,\sigma_2)$	$(-0.5\sigma_1,\sigma_2)$	$0.5(\sigma_1,0)$	$0.5(0,\sigma_2)$	$0.5(\sigma_1,\sigma_2)$
Root transformation	129820	54568	66487	18849	83651	65303	78399
NORTA inverse	129740	52690	62948	19500	81249	65003	70169
CCC & C	138720	84249	87253	42459	113680	94010	81450
Shift→	$(\sigma_1,0)$	$(0,\sigma_2)$	(σ_1,σ_2)	$1.5(\sigma_1,0)$	$1.5(0,\sigma_2)$	$1.5(\sigma_1,\sigma_2)$	$(2\sigma_1,0)$
Root transformation	59240	27260	42850	34559	12282	26078	28040
NORTA inverse	51310	25640	40790	34762	11348	22621	25170
CCC & C	80960	39580	44850	46260	18600	24320	36810
Shift→	$(0,2\sigma_2)$	$(2\sigma_1,2\sigma_2)$	$(3\sigma_1,0)$	$(0,3\sigma_2)$	$(3\sigma_1,3\sigma_2)$	$0.5(-\sigma_1,\sigma_2)$	$(\sigma_1,-\sigma_2)$
Root transformation	6140	14180	15460	1730	5720	20740	10290
NORTA inverse	5510	13770	15450	1640	4940	19010	10600
CCC & C	9710	15820	23540	2700	6230	42590	70290

As we are controlling a high quality process, *AIRL* (Average Item Run Length) is also reported for different methods in Table 6.

5.Numerical examples

5.2 Example 4

generate 10,000 observations

Table 7 ARL_1 values for different mean shifts in example 4

Shift→	$(-0.25\sigma_1,0,0)$	$(-0.25\sigma_1,\sigma_2,0)$	$(-0.25\sigma_1,0,\sigma_3)$	$(-0.25\sigma_1,\sigma_2,\sigma_3)$	$(-0.5\sigma_1,0,0)$	$(-0.5\sigma_1,\sigma_2,0)$	$(-0.5\sigma_1,0,\sigma_3)$	$(-0.5\sigma_1,\sigma_2,\sigma_3)$	$(\sigma_1,0,0)$
Root transformation	18.10	5.44	8.08	3.43	15.11	4.03	7.42	2.87	6.62
NORTA inverse	20.66	5.03	8.16	3.12	16.07	3.92	7.89	2.74	6.341
CCC & C	13.95	3.61	14.47	3.47	12.23	3.52	12.22	3.45	7.42
Shift→	$(0,\sigma_2,0)$	$(0,0,\sigma_3)$	$(\sigma_1,\sigma_2,0)$	$(\sigma_1,0,\sigma_3)$	$(0,\sigma_2,\sigma_3)$	$(\sigma_1,\sigma_2,\sigma_3)$	$2(\sigma_1,0,0)$	$2(0,\sigma_2,0)$	$2(0,0,\sigma_3)$
Root transformation	6.29	7.44	5.74	3.71	3.61	3.06	3.46	2.24	2.79
NORTA inverse	5.97	7.20	5.40	3.54	3.39	2.85	3.36	2.16	2.62
CCC & C	5.61	9.38	4.47	5.24	4.34	3.58	4.01	2.38	3.59
Shift→	$2(\sigma_1,\sigma_2,0)$	$2(\sigma_1,0,\sigma_3)$	$2(0,\sigma_2,\sigma_3)$	$2(\sigma_1,\sigma_2,\sigma_3)$	$3(\sigma_1,0,0)$	$3(0,\sigma_2,0)$	$3(0,0,\sigma_3)$	$3(\sigma_1,\sigma_2,0)$	$3(\sigma_1,0,\sigma_3)$
Root transformation	2.43	1.55	1.33	1.27	2.43	1.36	1.58	1.51	1.13
NORTA inverse	2.29	1.53	1.30	1.25	2.3957	1.34	1.55	1.48	1.13
CCC & C	2.09	2.21	1.66	1.53	2.8567	1.47	1.86	1.39	1.36
Shift→	$3(0,\sigma_2,\sigma_3)$	$3(\sigma_1,\sigma_2,\sigma_3)$	$(-\sigma_1,\sigma_2,0)$	$(\sigma_1,-\sigma_2,0)$	$(-\sigma_1,0,\sigma_3)$	$(\sigma_1,0,-\sigma_3)$	$(0,-\sigma_2,\sigma_3)$	$(0,\sigma_2,-\sigma_3)$	$(\sigma_1,-\sigma_2,-\sigma_3)$
Root transformation	1.03	1.03	3.97	2.50	7.91	5.00	3.98	4.46	2.05
NORTA inverse	1.03	1.02	3.87	2.66	7.55	4.93	4.60	4.56	2.10
CCC & C	1.12	1.10	3.468	7.67	12.59	5.98	11.30	4.72	6.14

Geometric probability distribution is p = 0.005.Poisson distribution with mean three(λ=4 and 5).

$$\hat{\Sigma} = \begin{pmatrix} 1 & 0.6 & -0.4 \\ 0.6 & 1 & -0.35 \\ -0.4 & -0.35 & 1 \end{pmatrix}.$$

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5.Numerical examples

Table 8 *AIRL* values for different mean shifts in example 4

Shift→	$(-0.25\sigma_1,0,0)$	$(-0.25\sigma_1,\sigma_2,0)$	$(-0.25\sigma_1,0,\sigma_3)$	$(-0.25\sigma_1,\sigma_2,\sigma_3)$	$(-0.5\sigma_1,0,0)$	$(-0.5\sigma_1,\sigma_2,0)$	$(-0.5\sigma_1,0,\sigma_3)$	$(-0.5\sigma_1,\sigma_2,\sigma_3)$	$(\sigma_1,0,0)$
Root transformation	2518.3	646.3	1067.9	359.7	2518.3	1431	291	657.8	189.4
NORTA inverse	2871.8	583.3	1081.4	307.6	2871.8	1497.6	276.7	716.6	172.4
CCC & C	1782.5	288	1847	277.4	1782.5	1057.1	193	1044.3	187.2
Shift→	$(0,\sigma_2,0)$	$(0,0,\sigma_3)$	$(\sigma_1,\sigma_2,0)$	$(\sigma_1,0,\sigma_3)$	$(0,\sigma_2,\sigma_3)$	$(\sigma_1,\sigma_2,\sigma_3)$	$2(\sigma_1,0,0)$	$2(0,\sigma_2,0)$	$2(0,0,\sigma_3)$
Root transformation	996.56	1234.4	1456.9	785.06	480.98	592.25	853.12	220.74	341.09
NORTA inverse	921.9	1180.3	1325.4	709.68	443.01	529.48	789.61	205.46	312.41
CCC & C	746.75	1606.4	932.35	1265.9	560.42	712.19	1012.1	179.74	538.38
Shift→	$2(\sigma_1,\sigma_2,0)$	$2(\sigma_1,0,\sigma_3)$	$2(0,\sigma_2,\sigma_3)$	$2(\sigma_1,\sigma_2,\sigma_3)$	$3(\sigma_1,0,0)$	$3(0,\sigma_2,0)$	$3(0,0,\sigma_3)$	$3(\sigma_1,\sigma_2,0)$	$3(\sigma_1,0,\sigma_3)$
Root transformation	509.25	163.67	60.69	85.01	522.78	60.14	111.87	189.69	36.71
NORTA inverse	447.57	151.45	53.62	74.69	484.06	56.91	104.65	173.89	37.57
CCC & C	307.91	444.76	94.981	167.41	668.05	46.64	201.21	105.83	149.04
Shift→	$3(0,\sigma_2,\sigma_3)$	$3(\sigma_1,\sigma_2,\sigma_3)$	$(-\sigma_1,\sigma_2,0)$	$(\sigma_1,-\sigma_2,0)$	$(-\sigma_1,0,\sigma_3)$	$(\sigma_1,0,-\sigma_3)$	$(0,-\sigma_2,\sigma_3)$	$(0,\sigma_2,-\sigma_3)$	$(\sigma_1,-\sigma_2,-\sigma_3)$
Root Transformation	5.70	9.36	288.45	383.43	711.06	1200.6	561.68	646.44	260.92
NORTA inverse	4.75	8.41	279.54	395.28	633.54	1154.7	646.38	655.84	259.12
CCC& C	15.20	33.73	190.60	1941.4	1122	1404.9	2029.6	588.35	1463.4

The results of Tables 7 and 8 show that the proposed methods perform well, especially in situations where there are both positive and negative shifts around the mean.

6.Conclusion and recommendations for future research

- 在多元計數的製程裡，計數之間的相關性在統計品質管制上是相當重要的議題。
- 在多元計數製程的監控，常未考慮到合格項目之間的獨立架構，故本文去主要在找第一個不合格項和不合格項裡的缺點數，然後用兩個資料轉換的方法：

1.Root transformation

2.NORTA inverse algorithm

再利用多元計值如χ^2 管制圖，去監控製程。

6.Conclusion and recommendations for future research

- 在本研究中，提出二個模擬的例子利用不同意同平均向量的移動，去比較三種方法(Root、NORTA、CCC&C)的ARL_1值。
- After the transformation phase, instead of χ^2 control chart we may want to examine other multivariate control charting techniques such as MEWMA and MCUSUM as well.

End