



Three-Quarter Replicates of 2^4 and 2^5 Designs

作者：John, P. W. M.

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指導教授：童超塵

報告學生：蔣天如



The 2^4 design

$$2^{4-1}$$

- Defined by $I = \pm ABCD$

主效應皆可估計

但二因子交互作用未被估計

- 使用 Three-Quarter Replicates



The 2^4 design

- 2^{4-2} Defined by $I = BD = ABC = ACD$

$$I = +BD = -ABC = -ACD ((1), ac, bcd, abd)$$

$$II = +BD = +ABC = +ACD (a, c, abcd, bd)$$

$$III = -BD = +ABC = -ACD (b, abc, cd, ad)$$

$$IV = -BD = -ABC = +ACD (ab, bc, acd, d)$$

- 刪除 I



The 2^4 design

- Combined into 2^{4-1}

$$II + III = +ABC$$

$$D = ABCD$$

$$AD = BCD$$

$$BD = ACD$$

$$CD = ABD$$

Effect	Sets Used	Confounding
A	III, IV; $x_4 = -x_2$	ABD
B	II, IV; $x_4 = x_1x_2$	ABCD
C	III, IV; $x_4 = -x_2$	BCD
D	II, III; $x_3 = x_1x_2$	ABCD
AB	II, IV; $x_4 = x_1x_2$	BCD
AC	III, IV; $x_4 = -x_2$	ABCD
AD	II, III; $x_3 = x_1x_2$	BCD
BC	II, IV; $x_4 = x_1x_2$	ABD
BD	II, IV; $x_4 = x_1x_2$	ABC
CD	or II, III; $x_3 = x_1x_2$	ACD
	II, III; $x_3 = x_1x_2$	ABD

- D, AD, BD, CD 被估計

The 2^4 design

- Combined into 2^{4-1}

$$II + IV = +ACD$$

$$B = ABCD$$

$$AB = BCD$$

$$BC = ABD$$

$$BD = ABC$$

Effect	Sets Used	Confounding
A	III, IV; $x_4 = -x_2$	ABD
B	II, IV; $x_4 = x_1x_2$	ABCD
C	III, IV; $x_4 = -x_2$	BCD
D	II, III; $x_3 = x_1x_2$	ABCD
AB	II, IV; $x_4 = x_1x_2$	BCD
AC	III, IV; $x_4 = -x_2$	ABCD
AD	II, III; $x_3 = x_1x_2$	BCD
BC	II, IV; $x_4 = x_1x_2$	ABD
BD	II, IV; $x_4 = x_1x_2$ or II, III; $x_3 = x_1x_2$	ABC ACD
CD	II, III; $x_3 = x_1x_2$	ABD

- B, AB, BC, BD 被估計



The 2^4 design

- Combined into 2^{4-1}

$$III + IV = BD$$

$$A = ABD$$

$$C = BCD$$

$$AC = ABCD$$

Effect	Sets Used	Confounding
A	III, IV; $x_4 = -x_2$	ABD
B	II, IV; $x_4 = x_1x_2$	ABCD
C	III, IV; $x_4 = -x_2$	BCD
D	II, III; $x_3 = x_1x_2$	ABCD
AB	II, IV; $x_4 = x_1x_2$	BCD
AC	III, IV; $x_4 = -x_2$	ABCD
AD	II, III; $x_3 = x_1x_2$	BCD
BC	II, IV; $x_4 = x_1x_2$	ABD
BD	II, IV; $x_4 = x_1x_2$ or II, III; $x_3 = x_1x_2$	ABC ACD
CD	II, III; $x_3 = x_1x_2$	ABD

- A, C, AC 被估計



The 2^4 design

$$3(2^{4-2})_V // 12$$

- Defined by $I = \pm BD = \pm ABC = \pm ACD$
- 實驗次數為12
- 主效應與二因子交互作用都可被估計
- 解析度為五的設計



The 2^4 design

$$2^{5-1}_V // 16$$

- Defined by $I = ABCDE$
- 主效應與二因子交互作用都可被估計
- 解析度為五的飽和（部分因子）設計

mate is $\sigma^2/4$, and all the degrees of freedom are used for effects. We present in this section a three-quarter replicate in which all the effects are estimable with variance $3\sigma^2/16$ and which provides 8 degrees of freedom for error.



The 2^5 design

- Davies (1956) $3(2^{5-2})_V // 24$
- Defined by $I = \pm BCE = \pm ABCD = \pm ADE$
- 主效應除了 E 都可被估計2次
- 二因子交互作用除了 AD, BC 都可被估計2次
- 解析度為五的設計 $\text{var} = \frac{3}{16} \sigma^2$



The 2^5 design

- Daniel (1958) $3(2^{5-2})_V // 24$
- Defined by $I = \pm AD = \pm BCE = \pm ABCDE$
- 主效應與二因子交互作用都可被估計2次
- 解析度為五的設計 $\text{var} = \frac{1}{4}\sigma^2$



參考文獻

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- [2] Davies, O. L., [1956]. *The Design and Analysis of Industrial Experiments*. Oliver and Boyd, London, 472-5.

■ THE END THANKS





發展

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TIME SEQUENCE FOR FACTORIAL EXPERIMENTS

PETER W.M.JOHN

Department of Mathematics, University of Texas, Austin, TX 78711
pwmj@math.utexas.edu

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