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Two Improved Runs Rules for the Shewhart X-bar Control Chart

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Abstract

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INTRODUCTION

1984年	Nelson	增加管制圖敏感度的法則
1987年	Champ and Woodall	提出以區間測試法來判別平均值跳動模型，並以馬可夫鏈來估算平均連串長度(ARL)作為評估管制法優劣之指標
1992年	Hurwitz and Mathur	提出了一個簡單的two-of-two rule控制 1.5δ。
2001年	Montgomery	這規則的使用，提高了管制圖檢測小變化的能力來，但卻大大增加Type-I error。
2000年	Klein	為了解決這個問題的退化控制平均運行長度 ARLo，提出了兩個強大的規則，一是指馬爾可夫鏈的方法，即在控制範圍可以調整所需ARLo價值。在本研究中，提出了兩種可供選擇的規則

- Klein2000年所建議的兩個規則為two-of-two rule and two-of-three rule，無論連續兩個百分點以上的(UCL)或連續兩次低於LCL都需要獲得out-of-control (o.o.c.) signal.
- Klein 2000年在檢測大的變化。為了克服這一弱點，建議提出了兩種可供選擇的規則以提高靈敏度. 兩個新的規則，首先是組合的規則之一one-of-one and two-of-two和one-of-one and two-of-three。如果足夠大的轉變一個一個規則只需要一個point就可獲得o.c.c。因此，這些新的規則在一個或兩個point可以發出信號o.c.c。這些規則將分別被稱為改進的two-of-two rule 和改進two-of-three rule。

- 兩種改進規則是改善two-of two rule and two-of-three rule改進two-of-two信號規則是一個o.o.c. 如果任何一個point超越界限UCL2 =LCL2或連續兩個point點超越界限UCL1=LCL1。o.o.c信號發出的改進two-ofthree rule如果任何一個point超越UCL2=LCL2，三點有連續二點超越界限UCL1=LCL1.

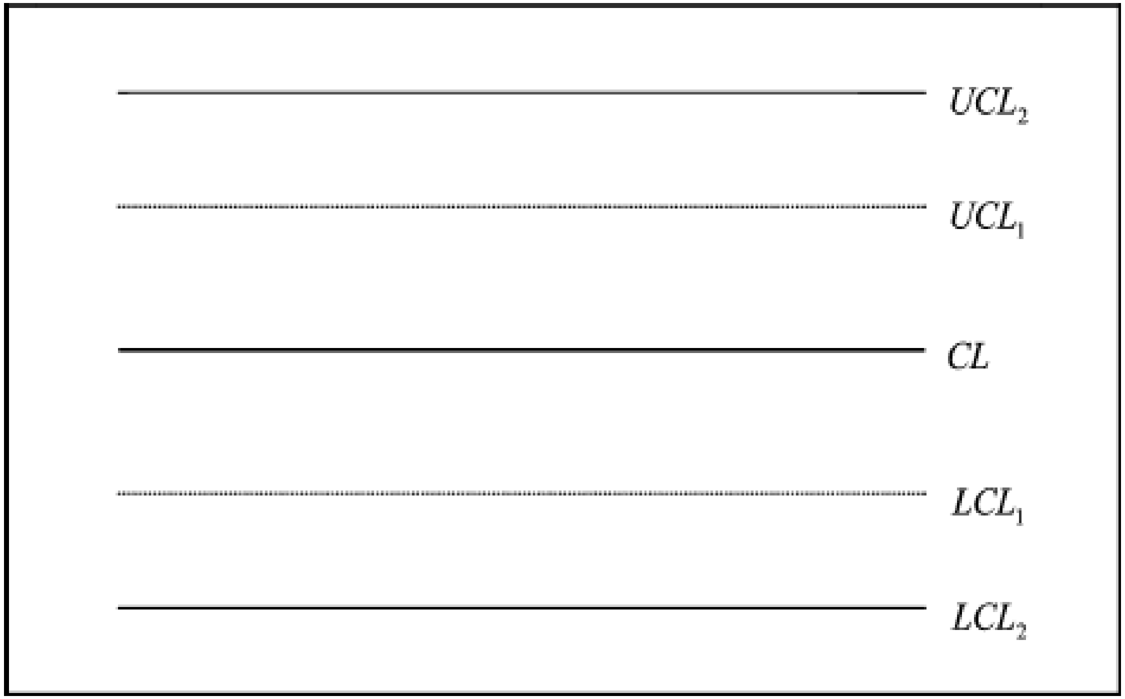

Figure 1. Control limits for a Shewhart $\overline{X}$ chart incorporating either the improved two-of-two or improved two-of-three rule.

- 控制界限Shewhart chart改進two-of-two or two-of-three rule.

Table 1
ARL profiles for the basic Shewhart control chart and that incorporating the two-of-two and the improved two-of-two rules based on $ARL_0 = 370.37$

			Improved two-of-two				
	Shewhart	Two-of-two	Two-of-two (A)	Two-of-two (B)	Two-of-two (C)	Two-of-two (D)	Two-of-two (E)
	Control limits						
Shift	± 3	± 1.7814	$UCL_1/LCL_1 = \pm 3.4$ & $UCL_2/LCL_2 = \pm 1.843$	$UCL_1/LCL_1 = \pm 3.5$ & $UCL_2/LCL_2 = \pm 1.823$	$UCL_1/LCL_1 = \pm 3.6$ & $UCL_2/LCL_2 = \pm 1.81$	$UCL_1/LCL_1 = \pm 3.7$ & $UCL_2/LCL_2 = \pm 1.798$	$UCL_1/LCL_1 = \pm 3.8$ & $UCL_2/LCL_2 = \pm 1.792$
0	371.20	375.48	373.22	373.25	373.84	369.57	372.69
0.2	311.60	280.52	281.48	281.57	282.24	277.16	278.38
0.4	204.13	153.98	156.50	155.63	155.78	153.88	153.58
0.6	120.22	81.14	81.97	81.39	80.83	80.04	80.00
0.8	71.86	45.33	45.30	44.90	44.77	44.75	44.65
1	44.28	26.95	26.40	26.40	26.45	26.37	26.38
1.2	28.38	16.99	16.38	16.42	16.51	16.45	16.55
1.4	18.03	11.25	10.68	10.68	10.75	10.75	10.85
1.6	12.51	7.92	7.48	7.48	7.48	7.52	7.60
1.8	9.00	5.83	5.41	5.45	5.48	5.48	5.52
2	6.39	4.52	4.14	4.17	4.21	4.24	4.27
2.2	4.72	3.71	3.32	3.34	3.38	3.41	3.45
2.4	3.65	3.19	2.80				
2.6	2.92	2.84	2.40				
2.8	2.36	2.58	2.12				
3	1.98	2.38	1.90				
4	1.19	2.04	1.27				
5	1.02	2.00	1.05				
6	1.00	2.00	1.01				

$$ARL_0 = \frac{1}{p} = \frac{1}{0.0027} = 370$$

使用任何改進的規則，先計算外部界 所需 ARL0值是370.37。
接下來，確定內部界限使用的UCL1 = LCL1公式源於一個Markov chain基礎上，UCL2 = LCL2已修復見附錄 A和B。請注意，限制，

Table 2

ARL profiles for the basic Shewhart control chart and that incorporating the two-of-three and the improved two-of-three rules based on $ARL_0 = 370.37$

			Improved two-of-three				
	Shewhart	Two-of-three	Two-of-three (A)	Two-of-three (B)	Two-of-three (C)	Two-of-three (D)	Two-of-three (E)
	Control limits						
Shift	± 3	± 1.9307	$UCL_1/LCL_1 = \pm 3.4$ & $UCL_2/LCL_2 = \pm 1.986$	$UCL_1/LCL_1 = \pm 3.5$ & $UCL_2/LCL_2 = \pm 1.966$	$UCL_1/LCL_1 = \pm 3.6$ & $UCL_2/LCL_2 = \pm 1.955$	$UCL_1/LCL_1 = \pm 3.7$ & $UCL_2/LCL_2 = \pm 1.946$	$UCL_1/LCL_1 = \pm 3.8$ & $UCL_2/LCL_2 = \pm 1.94$
0	371.20	374.48	373.13	371.06	372.94	369.96	369.33
0.2	311.60	277.11	278.78	275.61	276.46	275.39	275.03
0.4	204.13	145.00	148.85	145.41	145.25	144.66	143.90
0.6	120.22	74.47	76.57	75.16	74.59	74.24	73.80
0.8	71.86	40.89	41.57	41.08	40.86	40.66	40.47
1	44.28	23.80	24.24	23.85	23.78	23.59	23.55
1.2	28.38	15.32	15.31	15.07	15.05	15.04	15.08
1.4	18.03	10.23	10.13	10.01	9.98	10.01	10.03
1.6	12.51	7.24	7.05	7.00	7.01	7.02	7.04
1.8	9.00	5.38	5.52	5.14	5.16	5.16	5.17
2	6.39	4.28	4.27	4.00	4.03	4.05	4.09
2.2	4.72	3.54	3.45	3.26	3.31	3.33	3.36
2.4	3.65	3.06	2.94	2.78	2.81	2.84	2.87
2.6	2.92	2.73	2.58	2.40	2.44	2.48	2.51
2.8	2.36	2.50	2.28	2.14	2.19	2.22	2.25
3	1.98	2.35	2.07	1.94	1.98	2.02	2.06
4	1.19	2.03	1.42	1.31	1.34	1.38	1.43
5	1.02	2.00	1.11	1.06	1.07	1.09	1.11
6	1.00	2.00	1.01	1.01	1.01	1.01	1.01

EVALUATING THE PERFORMANCES OF THE IMPROVED RULES

- 模擬研究，採用 SAS，版本6.12，評價的ARL性能的改進規則， Klein 2000年提出的兩個規則。每個的ARL值計算基於5000仿真試驗。其ARL值在標準Shewhart three-sigma 管制圖中也被算出。對於所有的計算，假定該points於管制圖上是獨立且同分配之標準常態分配。製程平均數從(控制內)至o.o.c.值高達6six sigma. 改進後的ARL值370.37，相當於型I誤差0.0027。
- 對於每一個改進的規則，考慮5例不同UCL2=LCL2 (i.e.3.4, 3.5, 3.6, 3.7,and 3.8) 五個案被稱為A, B, C, D, and E, 分別在表1和2。該 UCL1= LCL1值改進two-of-two rule 對應於這些 UCL2= LCL2限制使用公式確定在附錄 A是分別是1.843, 1.823, 1.81, 1.798, and1.792, 同樣，價值 UCL1= LCL1計算的規則，使用改進的two-ofthree附錄 B中的公式，對應於上述UCL2= LCL2值，分別是1.986, 1.966, 1.955, 1.946, and1.94,

- 表1包含two-of-two rule的的ARL結果，而表2顯示結果為改進two-of-three規則。
- 兩個表中的結果清楚地表明，平均而言，兩種改進的規則將信號o.o.c.關於第一點後移的製程中平均變化幅度大five or six sigma
- 這是一個改善Klein 2000年提出的與這兩個規則在最快的時間一o.o.c.信號可以被檢測到的檢測是在第二個點後移。對於非常小中等的變化，ARL改進規則與Klein (2000) 具有可比性。因此，對於Klein (2000)年所提出的改進two-of-two and two-of-three rules是很好的選擇，
- 還要注意的是，標準休哈特 管制圖與 3 sigma的界限有表現最差。

CONCLUSION

- 改進後的two-of-two and two-of-three規則，有較好的ARL性能比Klein在2000年提出的相應的規則檢測大的變化，同時保持同樣的優勢檢測小到中等製程平均值變化。一般廣泛時應當謹慎，同時使用兩種運行規則，對所有的型I誤差將不會大幅增加
- 基本上一種改進的兩個規則，即由two-of-two and one-of-one規則，這裡的原則是前規則的變化迅速作出反應小，而後者則提供了一種快速o.o.c.信號大的變化。改進後的two-of-three其原理是相同的。

APPENDIX A

- 在下面的討論中，被認為是控制圖組成的範圍如圖 1所示。

In the following discussion, a control chart is viewed as consisting of the limits shown in Figure 1. Denote the probability of a single point falling between LCL_1 and UCL_1 by r, between UCL_1 and UCL_2 by r_U, between LCL_1 and LCL_2 by r_L, above UCL_2 by r_{2U}, and below LCL_2 by r_{2L}. The values of these probabilities determine the location of the control limits. The following absorbing Markov chain with three transient states and an absorbing state is considered:

State {1}: no points beyond any of the control limits
State {2}: a point between UCL_1 and UCL_2
State {3}: a point between LCL_1 and LCL_2
State {4}: the absorbing state, when either a point is beyond the UCL_2/LCL_2 limits, or if two successive points fall between UCL_1 and UCL_2 or between LCL_1 and LCL_2

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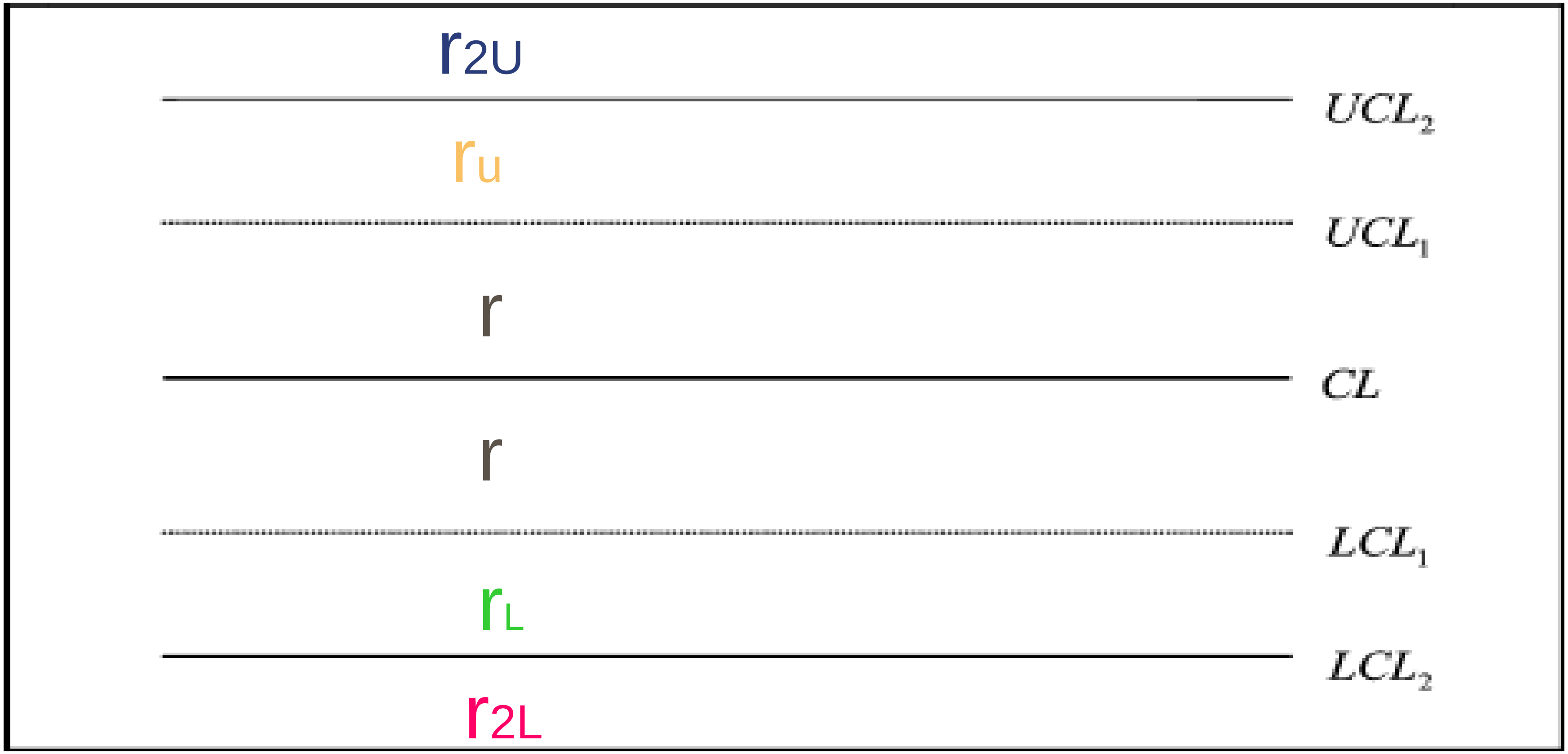

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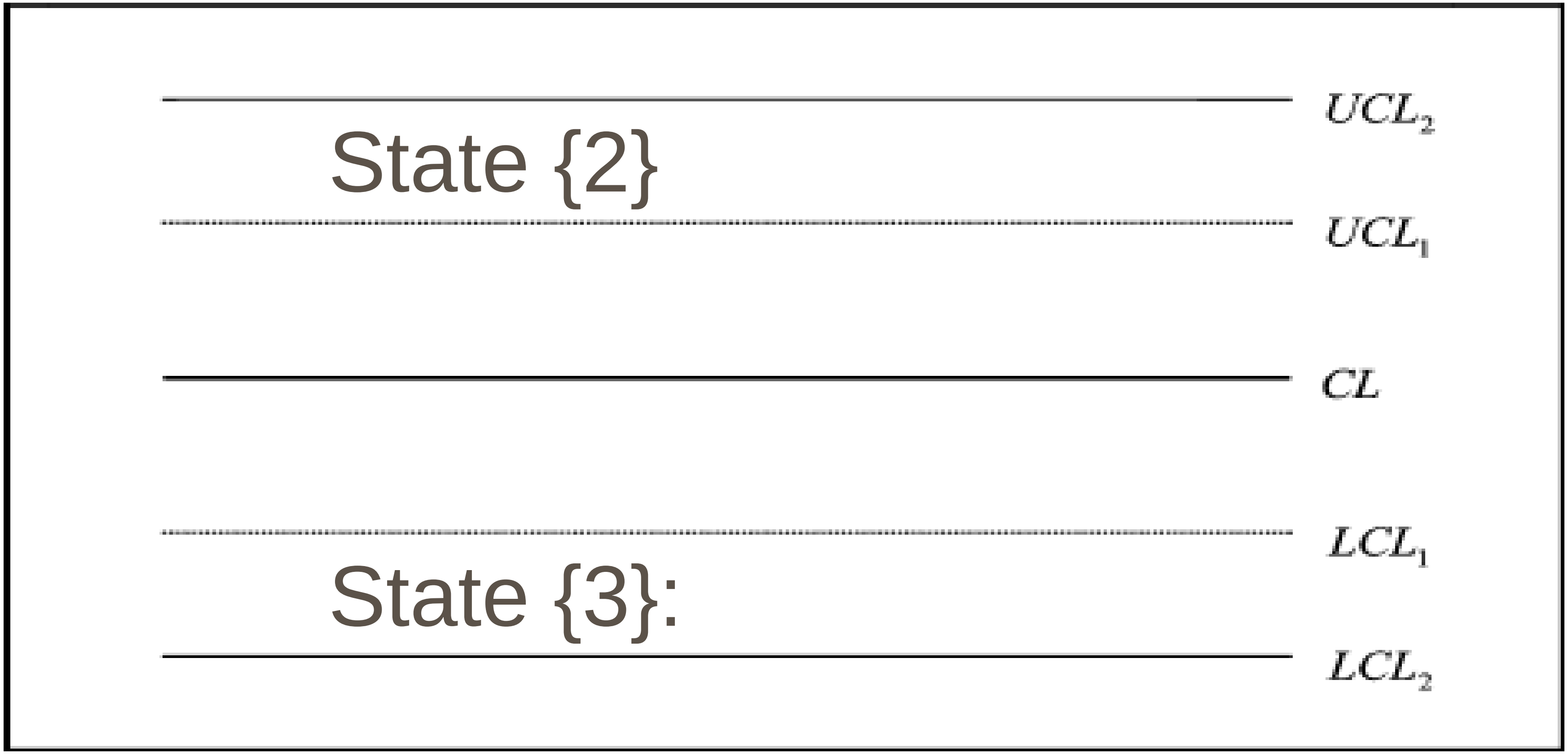

- The transition probabilities of the Markov chain are given in Table 3.

Table 3
Transition probabilities of the improved two-of-two rule for Markov chain with three transient states

States at time t	States at time $t+1$			
	1	2	3	4
1	r	r_U	r_L	$r_{2U} + r_{2L}$
2	r	0	r_L	$r_U + r_{2U} + r_{2L}$
3	r	r_U	0	$r_L + r_{2U} + r_{2L}$
4	0	0	0	1

$$\begin{aligned} N_{14} &= 1 + (r)N_{14} + (r_U)N_{24} + (r_L)N_{34} \\ N_{24} &= 1 + (r)N_{14} + (r_L)N_{34} \\ N_{34} &= 1 + (r)N_{14} + (r_U)N_{24} \end{aligned} \tag{1}$$

Because symmetric control limits are used, let $r_{2U} = r_{2L} = a$ and $r_U = r_L = b$. Due to the fact that $r + r_U + r_L + r_{2U} + r_{2L} = 1$, and from the previous information, it is easily shown that the in-control ARL is

$$\mathrm{ARL}_0 = N_{14} = \frac{1+b}{2(a+ab+b^2)}. \tag{2}$$

To illustrate finding the limits of the improved two-of-two rule, consider the following example: Assume that for a process that follows a normal distribution, we set $\mathrm{ARL}_0 = 370.37$ and the outer limits, $UCL_2/LCL_2 = \pm 3.5\sigma$. From the standard normal table, it is found that $a = 0.000233$. Substituting the values of a and ARL_0 into Eq. (2) and solving for b gives $b = 0.0340043$. Finding the inner limits using a standard normal table gives $UCL_1/LCL_1 = \pm 1.823\sigma$.

APPENDIX B

- Markov chain with seven transient states and an o.o.c. state. The states description are

State (OO): has two successive points between UCL_1 and LCL_1

State (OU): has a first point between UCL_1 and LCL_1, and the second between UCL_1 and UCL_2

State (OL): has a first point between UCL_1 and LCL_1, and the second between LCL_1 and LCL_2

State (UL): has a first point between UCL_1 and UCL_2, and the second between LCL_1 and LCL_2

State (UO): has a first point between UCL_1 and UCL_2, and the second between UCL_1 and LCL_1

State (LO): has a first point between LCL_1 and LCL_2, and the second between UCL_1 and LCL_1

State (LU): has the first point between LCL_1 and LCL_2, and the second between UCL_1 and UCL_2

State (OOC): the absorbing state, when either a point is beyond the UCL_2/LCL_2 limits, or if two of three successive points fall between UCL_1 and UCL_2 or between LCL_1 and LCL_2

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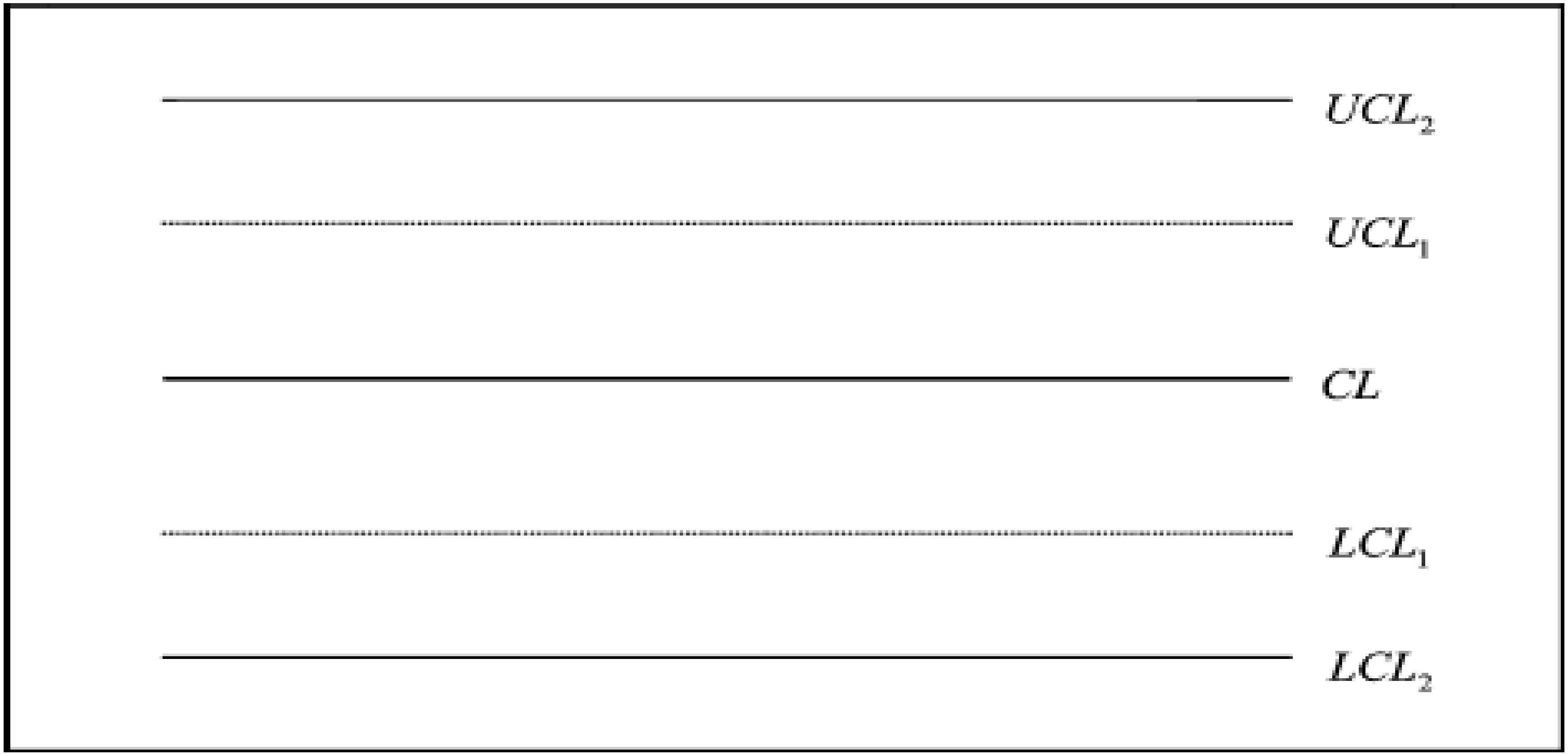

The transition probabilities of the Markov chain are given in Table 4.

The in-control ARL is given by the expected value of the first passage time from the starting state 1 to the o.o.c. state 8. To obtain the expected first passage times from each state to the absorbing state, we need to solve the following linear system:

$$
\begin{aligned}
N_{18} &= 1 + (r)N_{18} + (r_U)N_{28} + (r_L)N_{38} \\
N_{28} &= 1 + (r)N_{58} + (r_L)N_{48} \\
N_{38} &= 1 + (r)N_{68} + (r_U)N_{78} \\
N_{48} &= 1 + (r)N_{68} \\
N_{58} &= 1 + (r)N_{18} + (r_L)N_{38} \\
N_{68} &= 1 + (r)N_{18} + (r_U)N_{28} \\
N_{78} &= 1 + (r)N_{58}
\end{aligned} \tag{3}
$$

Table 4

Transition probabilities of the improved two-of-three rule for Markov chain with seven transient states

	States at time $t+1$							
States at time t	(OO)	(OU)	(OL)	(UL)	(UO)	(LO)	(LU)	(OOC)
(OO)	r	r_U	r_L					$r_{2U}+r_{2L}$
(OU)				r_L	r			$r_U+r_{2U}+r_{2L}$
(OL)						r	r_U	$r_L+r_{2U}+r_{2L}$
(UL)						r		$r_U+r_L+r_{2U}+r_{2L}$
(UO)	r		r_L					$r_U+r_{2U}+r_{2L}$
(LO)	r	r_U						$r_L+r_{2U}+r_{2L}$
(LU)					r			$r_U+r_L+r_{2U}+r_{2L}$
(OOC)								1

$a =$

$\text{ARL0} =$

$$\text{NSolve} = \left[\frac{-1+(-3+2a)b+(-1+2a)b^2+2b^3}{2(2a^2b(1+b)+b^2(-2-b+2b^2)+a(-1-3b+b^2+4b^3))} = \text{ARL0},\ b\right]$$

Figure 2. A Mathematica 4.0 program to compute the value of *b*.

Similar to the improved two-of-two rule, the additional constraints are $r_{2U} = r_{2L} = a$, and $r_U = r_L = b$, and $r + r_U + r_L + r_{2U} + r_{2L} = 1$. Using Mathematica 4.0 to solve the linear system in Eq. (3) with these additional constraints for the in-control ARL give

$$\mathrm{ARL}_0 = N_{18} = \frac{-1 + (2a-3)b + (2a-1)b^2 + 2b^3}{2[2a^2b(1+b) + b^2(2b^2 - b - 2) + a(4b^3 + b^2 - 3b - 1)]} \quad (4)$$

To construct a Shewhart chart based on a desired in-control ARL value incorporating the improved two-of-three rule, first we need to fix the outer UCL_2/LCL_2 limits. Then, using a standard normal table, determine the value of a. Next, find the corresponding value of b using the "Nsolve" procedure given in Mathematica 4 (Figure 2).

Input the values of a and ARL_0 into the program in Figure 2 where ARL_0 represents the desired in-control ARL. Running this simple program by pressing the "shift" and "enter" keys simultaneously gives the value of b. Using a standard normal table, the limits UCL_1/LCL_1 can be determined.

END