Accelerated Life Testing On Repairable Systems

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SUMMARY & CONCLUSIONS

In this paper, we define two accelerated life models for repairable systems: the Arrhenius-exponential model and the Peck-Weibull model. Thus, we show that is possible to estimate the reliability of a product during its development with a small number of prototypes using accelerated life testing with the ability to repair when a failure occurs. This method allows us to improve the accuracy in the estimation of reliability parameters where the accuracy is linked to the number of failure times that are available. Nevertheless, these models assume "minimal repairing" such that any repair has no impact on the failure rate.

1. *INTRODUCTION*

Industrial competitiveness in terms of innovation, time of development, and field reliability expectations leads to more efficient strategies to mature a product. In particular, engineers are looking for methods to evaluate the reliability, as cheaply as possible, knowing that the market demands continuously improvement in system reliability. This leads to longer duration testing that may be incompatible with industrial constraints. In order to reduce test time, we can use accelerated life testing. In these tests, systems are tested under a higher level of usage and/or environmental stresses in order to accelerate the failure mechanisms (assumed to be the same ones as those in nominal conditions) and so reduce the test time required to estimate behavioural characteristics of the product in nominal conditions. (see figure 1).

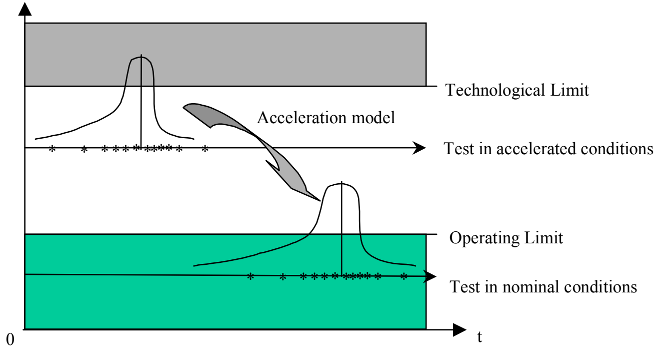

Figure 1 : Reliability assessment with accelerated tests

Accelerating life testing solves this first aspect of the duration but there is an other problem. To evaluate the reliability during the development of a new product, we use prototypes which are expensive and therefore we have few specimens. In many cases, failures seen at the system level are due to the failure of one component (serial system). Thus, it is possible to repair (to change the failed component) and continue testing the system. (ref 4). For example, an electronic board where the failure can be induced by a capacitor failure can be easily repaired and testing resumed.

In this paper we propose utilization of an Accelerated Life Testing (ALT) model for repairable systems. It's necessary to know :

- the analytic model which links the degradation rate to the severity and amplitude of the system usage.
- the parameter values for the controlling variable of the chosen lifetime models, such as the activation energy in the Arrhenius model for example.

2. *USUAL LIFETIME DISTRIBUTIONS, TEST PLANS WITH REPAIRING AND ACCELERATION MODEL*

2.1 *Common lifetime distributions*

In an accelerated life testing, the behaviour of the product lifetime is not described simply by one relationship. For each stress level, the system is characterized by a statistical distribution of lifetimes. So we must combine an acceleration model and a lifetime distribution. We are going to present some of the usual distributions obtained with accelerated lifetime tests common to several fields. Some of their main properties, associated to reliability functions and failure rates, will be mentioned (ref. 1, 10).

2.1.1 Exponential distribution

This distribution models many applications in several fields. It is a simple distribution, very common in reliability where the failure rate is constant. It relates the lifetime of equipment characterized by random failures. The reliability function of an exponential distribution with a θ parameter is :

$$R(t) = e^{-\frac{t}{\theta}} \qquad [1.]$$

Consequently, the failure rate is :

$$\lambda(t)=\frac{1}{\theta} \qquad [2.]$$

2.1.2 –Weibull distribution

A very popular distribution, applied in electronics as well as in mechanics and is a more accurate model for the behaviour of a product during the three usual stages of its life: infant mortality with a decreasing failure rate, constant failure rate, and wearout period with increasing failure rate. The reliability function of a Weibull distribution with η and β parameters is :

$$R(t)=e^{\left(-\frac{t}{\eta}\right)^{\beta}} \qquad [3.]$$

The failure rate is:

$$\lambda(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \qquad [4.]$$

2.2 Test plans with repairing

Suppose successive times to failure for a system (characterized by a failure rate λ(t)) are observed (ref 2-4, 10). The system is new at the initial time. When a failure occurs, the system is immediately repaired. The repair is assumed to be "minimal". This means that the system functions properly but its failure rate is not modified. We consider a test plan with a fixed test time τ. During the test, n systems are placed under test where any failure is repaired immediately. The system i is observed until time τ, and the number of observed failures is K_i (random variable) and the times to failure are T_j^i $(0 \le j \le K_i)$ (see Figure 2).

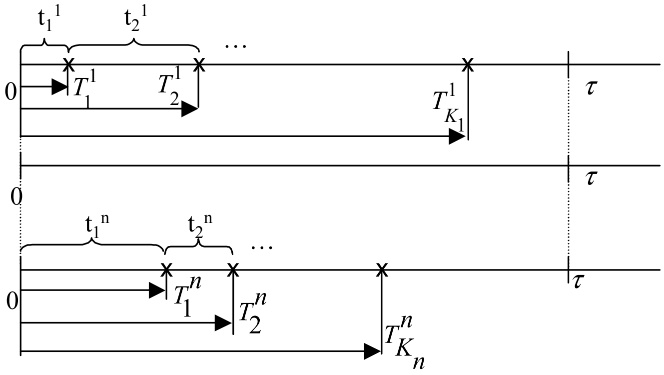

Figure 2 : Record of failure times

Intuitively, the failure-process intensity (for a repairable system) is a measure of the probability that a failure occurs shortly after *T*. The failure-process intensity can be a function of both the running time *T* and the local time *t* (see Figure 2). If the failure-process intensity is a constant, λ, there is a Homogeneous Poisson Process (HPP). This means that the times between failures are statistically independent and identically distributed with parameter λ.

When the failure-process intensity is a function only of the running time *T*, the failures can occur according to a Non-Homogeneous Poisson Process (NHPP). This situation is obtained if, for instance, we use a minimal repair policy, i.e., the unit is repaired to the state of the system just before the failure. One of the most popular NHPP models is the Weibull process model.

2.2.1 Case of Homogeneous Poisson process

The observation of system i is defined by homogeneous Poisson process with stochastic intensity λ in time interval [0,τ] (ref 2-4, 10). The log-likelihood is written :

$$L(\lambda)=\left(\sum_{i=1}^{n} K_i\right)\log(\lambda)-n\lambda\tau \qquad [5.]$$

The estimator of failure rate λ is given by :

$$\hat{\lambda}=\frac{\sum_{i=1}^{n} K_i}{n\tau} \qquad [6.]$$

2.2.2 Case of non-homogeneous Poisson process

In this section, a non-homogeneous Poisson process (with parameters β and η) is defined with stochastic intensity :

$$\lambda(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \qquad [7.]$$

This process defines the times to failure of a system characterized by a Weibull distribution and which is subject to "minimal repairing" (ref 2-4, 10). The log-likelihood is written :

$$L(\beta,\eta)=(\log\beta-\beta\log\eta)\left(\sum_{i=1}^{n} K_i\right)+(\beta-1)\sum_{i=1}^{n}\sum_{j=1}^{K_i}\log T_j^i-n\left(\frac{\tau}{\eta}\right)^{\beta} \qquad [8.]$$

The maximum likelihood estimators β and η are deducted from :

$$\frac{1}{\hat{\beta}}=\log\tau-\frac{1}{\sum_{i=1}^{n} K_i}\sum_{i=1}^{n}\sum_{j=1}^{K_i}\log\left(T_j^i\right) \qquad [9.]$$

$$\log\hat{\eta}=\log\tau+\frac{1}{\hat{\beta}}\log(n)-\frac{1}{\hat{\beta}}\log\left(\sum_{i=1}^{n} K_i\right) \qquad [10.]$$

2.3 *Usual acceleration models*

2.3.1 Arrhenius Model

This model is used when the failure mechanism is driven by temperature (especially for dielectrics, semi-conductors, battery cells, lubricant, grease, plastic, incandescent filaments). The Arrhenius model defines the lifetime τ of the product as a function of temperature (ref 1, 5, 6, 7, 8, 9) :

$$\tau=Ae^{\left(\frac{+E_a}{kT}\right)} \qquad [11.]$$

With : A positive constant
E_a activation energy in eV
k Boltzman's constant (8.6171 10^{-5} eV/K)
T absolute temperature

The Arrhenius acceleration factor between the lifetime τ_1 for a temperature T_1 and the lifetime τ_2 for a temperature T_2 is :

$$FA=\frac{\tau_1}{\tau_2}=e^{\frac{Ea}{k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)} \qquad [12.]$$

2.3.2 Peck Model

This model is used where the failure mechanism is driven by temperature and humidity (especially for electrical components, aluminium conductors and mechanical components submitted to breaking).

The Peck model defines the degradation rate with a temperature T and a humidity level H given by the following relationship (ref 1, 5, 6, 7, 8, 9) :

$$\tau = A(H)^{-m} e^{\left(\frac{E_a}{kT}\right)} \qquad [13.]$$

with : T : absolute temperature (in °K)
H : humidity in %
k : 8.6171 10^{-5} eV/°K Boltzmann constant
E_a : activation energy in eV
m : constant

3. *ACCELERATED LIFE MODEL ON REPAIRABLE SYSTEMS*

3.1 *Application to exponential distribution coupled with Arrhenius*

In this section, an exponential lifetime distribution with an Arrhenius acceleration model will be presented. For that purpose, it is considered that :

- The lifetime is defined by an exponential distribution
- The test plan is defined in section 2.2.1
- The scale parameter λ of the exponential distribution is defined by an Arrhenius model :

$$\lambda_i = \lambda_0 e^{\left(-\frac{E_a}{k}\left(\frac{1}{T_i}-\frac{1}{T_0}\right)\right)} \qquad [14.]$$

with $\lambda_i = \frac{1}{\tau_i}$ and T_0 the nominal temperature

If it is posed :

$$x_i = -\frac{1}{k}\left(\frac{1}{T_i}-\frac{1}{T_0}\right) \qquad [15.]$$

Then, equation [14] becomes :

$$\lambda_i = \lambda_0 \, e^{\left(E_a x_i\right)} \qquad [16.]$$

Which is the classical COX model.

In order to define the model, the two unknown parameters E_a and λ_0 have to be evaluated. For this purpose, two tests are realised under two temperatures (T_1 and T_2). At each temperature level, the values x_i and λ_i are estimated :

x_1 and x_2 by relationship [15]
λ_1 and λ_2 by relationship [6]

The parameters λ_0 and E_a are estimated from relationship [16] in :

$$E_a = \frac{1}{x_1 - x_2} log\left(\frac{\lambda_1}{\lambda_2}\right) \qquad [17.]$$

$$ln(\lambda_0) = \frac{x_2 ln(\lambda_1) - x_1 ln(\lambda_2)}{x_2 - x_1} \qquad [18.]$$

Numerical example :

For this example, the simulation parameters are :

- Ea = 0.37 eV (activation energy)
- N= 5 (sample size by stress level)
- λ_0 = 5* 10^{-5} h^{-1} (baseline failure rate)
- τ = 1000 h (censored time)
- T_0 = 30°C (nominal temperature)
- T_1 = 120°C (temperature for test 1)
- T_2 = 200°C (temperature for test 2)

The test results are obtained by simulation (in drawing on cdf (1-R)) with these parameters (see Table1 and Table 2).

The failure rates λ_1 and λ_2 are obtained in applying the Arrhenius model (eq [14]) :

$$\lambda_1 = \lambda_0 e^{\left(-\frac{E_a}{k}\left(\frac{1}{T_1}-\frac{1}{T_0}\right)\right)}$$

$$\lambda_1 = 5\times10^{-5} e^{\left(-\frac{0.37}{8.6171\times10^{-5}}\left(\frac{1}{273+120}-\frac{1}{273+30}\right)\right)} = 1.28\times10^{-3} h^{-1}$$

$$\lambda_2 = \lambda_0 e^{\left(-\frac{E_a}{k}\left(\frac{1}{T_2}-\frac{1}{T_0}\right)\right)}$$

$$\lambda_2 = 5\times10^{-5} e^{\left(-\frac{0.37}{8.6171\times10^{-5}}\left(\frac{1}{273+200}-\frac{1}{273+30}\right)\right)} = 8.15\times10^{-3} h^{-1}$$

The local failure times are simulated by :
Temperature T_1 = 120°C : $t_i = F(t)^{-1}$ with $F(t) = 1\text{-}e^{-\lambda_1 t}$
Temperature T_2 = 200°C : $t_i = F(t)^{-1}$ with $F(t) = 1\text{-}e^{-\lambda_2 t}$
The absolute times are deduced by :

$$T_i = \sum_{j=1}^{i} t_j$$

Table 1 : Simulated test results with temperature T=120°C

System index	Time to failure T_i (in hours) and censored time in bold			Failures number n
1	397	872	**1000**	2
2	**1000**			0
3	646	**1000**		1
4	850	851	**1000**	2
5	69	**1000**		1

The values x_1 and λ_1 are estimated (by [15] and [6]) :

x_1 = 8.788
λ_1 =1.2* 10^{-3} h^{-1} (instead of 1.28* 10^{-3} h^{-1})

The values x_2 and λ_2 are estimated (by [15] and [6]) :
x_2 =13.76
λ_2 = 7.4* 10^{-3} h^{-1} (instead of 8.15* 10^{-3} h^{-1})

The Arrhenius (E_a) and exponential (λ_0) parameters can be estimated by relationships [17] and [18]. Thus, the precision of these values can be improved based upon test data. :

- E_a = 0. 365 eV (instead of 0.37 eV)
- λ_0 =4.85* 10^{-5} h^{-1} (instead of 5* 10^{-5} h^{-1})

It can be noted that the estimators values are close to initial data.

Table 2 : Simulated test results with temperature T=200°C

System index	Time to failure T_i (in hours) and censored time in bold													n
1	329	345	349	393	509	922	**1000**							6
2	39	79	109	189	502	551	730	**1000**						7
3	52	170	849	**1000**										3
4	175	206	223	246	268	288	302	543	545	612	663	848	**1000**	12
5	79	234	337	600	650	710	735	971	994	**1000**				9

3.2 *Application to Weibull distribution in considering an Peck acceleration model*

In this section, the accelerated life Weibull model will be used for step stress with a Peck acceleration model. For that purpose, it is considered that :

- The lifetime is defined by a Weibull distribution
- The test plan is defined in paragraph 2.2.2
- The shape parameter β of the Weibull distribution is constant
- The scale parameter η of the Weibull distribution is defined by a Peck model :

$$\eta(T,H)=AH^{-m}e^{\frac{E_a}{kT}} \qquad [19.]$$

In order to define the model, we have to evaluate A, m and E_a. For this purpose, three tests are performed with different temperature and humidity levels. The unknown variables are estimated by identification of terms between relations [10] and [19].

Numerical example :

For this example, the simulation parameters are :

- E_a = 0.51 eV (activation energy)
- n= 5 (sample size by stress level)
- m = 6.2
- k = 8.6171* 10^{-5} eV/K Boltzmann's constant
- A = 8.75 10^5 days
- β = 1.5
- T_1 = 65°C (Temperature for test 1)
- T_2 = 85°C (Temperature for test 2)
- T_3 = 85°C (Temperature for test 3)
- H_1 = 90% (Humidity for test 1)
- H_2 = 90% (Humidity for test 2)
- H_3 = 95% (Humidity for test 3)

The test results are obtained by simulation (in drawing on cdf (1-R)) with these parameters (see table3, table 4 and table 5). The scale parameters η_1, η_2 and η_3 are obtained in applying the Peck model (eq [19]) :

$$\eta_1=AH_1^{-m}e^{\frac{E_a}{kT_1}}$$

$$\eta_1=8.75\times10^5 90\%^{-6.2}e^{\frac{0.51}{8.6171\times10^{-5}(273+65)}}=26.93\,days$$

$$\eta_2 = A \quad H_2^{-m} \quad e^{\frac{E_a}{kT_2}}$$

$$\eta_2 = 8.75\times10^5 \quad 90\%^{-6.2} \quad e^{\frac{0.51}{8.6171\times10^{-5}(273+85)}} = 10.13 \quad days$$

$$\eta_3=AH_3^{-m}e^{\frac{E_a}{kT_3}}$$

$$\eta_3=8.75\times10^5 95\%^{-6.2}e^{\frac{0.51}{8.6171\times10^{-5}(273+85)}}=7.24\,days$$

The failure times T_i are simulated by this method (see Figure 3) :

$T_1 = F(t)^{-1}$ with $F(t)=1-e^{-\left(\frac{t}{\eta}\right)^\beta}$ and $F(t)\in[0,1]$

$T_2 = F(t)^{-1}$ with $F(t)=1-e^{-\left(\frac{t}{\eta}\right)^\beta}$ and $F(t)\in[F_1,1]$ where $F_1=F(T_1)$

…

$T_i = F(t)^{-1}$ with $F(t)=1-e^{-\left(\frac{t}{\eta}\right)^\beta}$ and $F(t)\in[F_{i-1},1]$ where $F_{i-1}=F(T_{i-1})$

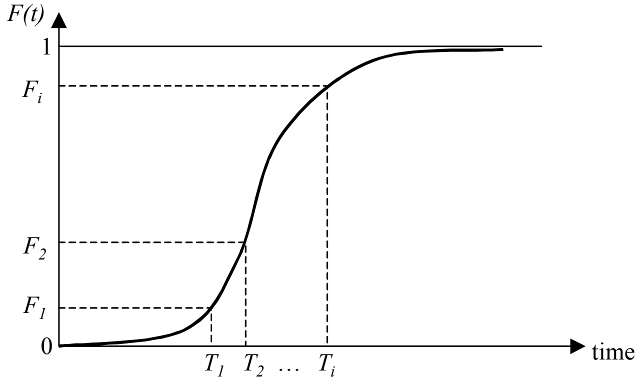

Figure 3 : Failure times simulation

Table 3 : Simulated test results with T=65°C and H=90%

	Time to failure (in days) and censored time in bold		
System 1	4.83	6.88	**20.00**
System 2	**20.00**		
System 3	6.59	**20.00**	
System 4	**20.00**		
System 5	13.81	16.52	**20.00**

The Weibull parameters η_1 and β_1 are estimated by the relationships [9] and [10] :

$\beta_1 = 1.20$ (instead of 1.5)

$\eta_1 = 20.00$ days (instead of 26.93 days estimated by [19])

Table 4 : Simulated test results with T=85°C and H=90%

	Time to failure (in days) and censored time in bold			
System 1	6.87	7.98	9.28	**10.00**
System 2	1.34	**10.00**		
System 3	1.25	6.00	**10.00**	
System 4	**10.00**			
System 5	**10.00**			

The Weibull parameters η_2 and β_2 are estimated by the relationships [9] and [10] :

$\beta_2 = 1.14$ (instead of 1.5)

$\eta_2 = 8.52$ days (instead of 10.12 days estimated by [19])

Table 5 : Simulated test results with T=85°C and H=95%

	Time to failure (in days) and censored time in bold				
System 1	4.99	8.67	**10.00**		
System 2	9.15	**10.00**			
System 3	5.86	5.90	8.06	8.20	**10.00**
System 4	0.88	2.28	9.00	**10.00**	
System 5	**10.00**				

The Weibull parameters η_3 and β_3 are estimated by the relationships [9] and [10] :

$\beta_3 = 1.56$ (instead of 1.5)

$\eta_3 = 6.41$ days(instead of 7.24 days estimated by [19])

In order to evaluate the activation energy, the ratio of η defined for the test 1 and 2 can be evaluated :

$$E_a = \frac{k \ln\left(\frac{\eta_1}{\eta_2}\right)}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} \qquad [20.]$$

Thus $E_a = 0.44$ eV (instead of 0.51 eV) To evaluate m, the ratio of the η defined in test 2 and 3 can be calculated :

$$m = \frac{-\ln\left(\frac{\eta_2}{\eta_3}\right)}{\ln\left(\frac{H_2}{H_3}\right)} \qquad [21.]$$

Thus m = 5.26 (instead of 6.2)

And, to close with, A can be evaluated from the first test :

$$A = \frac{\eta_1}{H_1^{-n} e^{\frac{E_a}{kT_1}}} \qquad [22.]$$

Thus $A = 1.19 * 10^5$ days (instead of $8.75*10^5$)

It can be noted that the estimators values are close to initial data.

4. *CONCLUSION*

In our work, we have defined two accelerated life models for repairable systems (Arrhenius-exponential model and Peck-Weibull model). Thus, we show that is possible to estimate the reliability of a product during its development (with a small number of prototypes using accelerated life tests and repairing when a failure occurs). This method is able to improve the accuracy in the estimation of reliability parameters (accuracy is linked to the number of failure times which are available). Nevertheless, these models assume “minimal repairing” such that there is no impact on the failure rate. Future work will be focused on the study of the impact of repair on the failure rate.

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